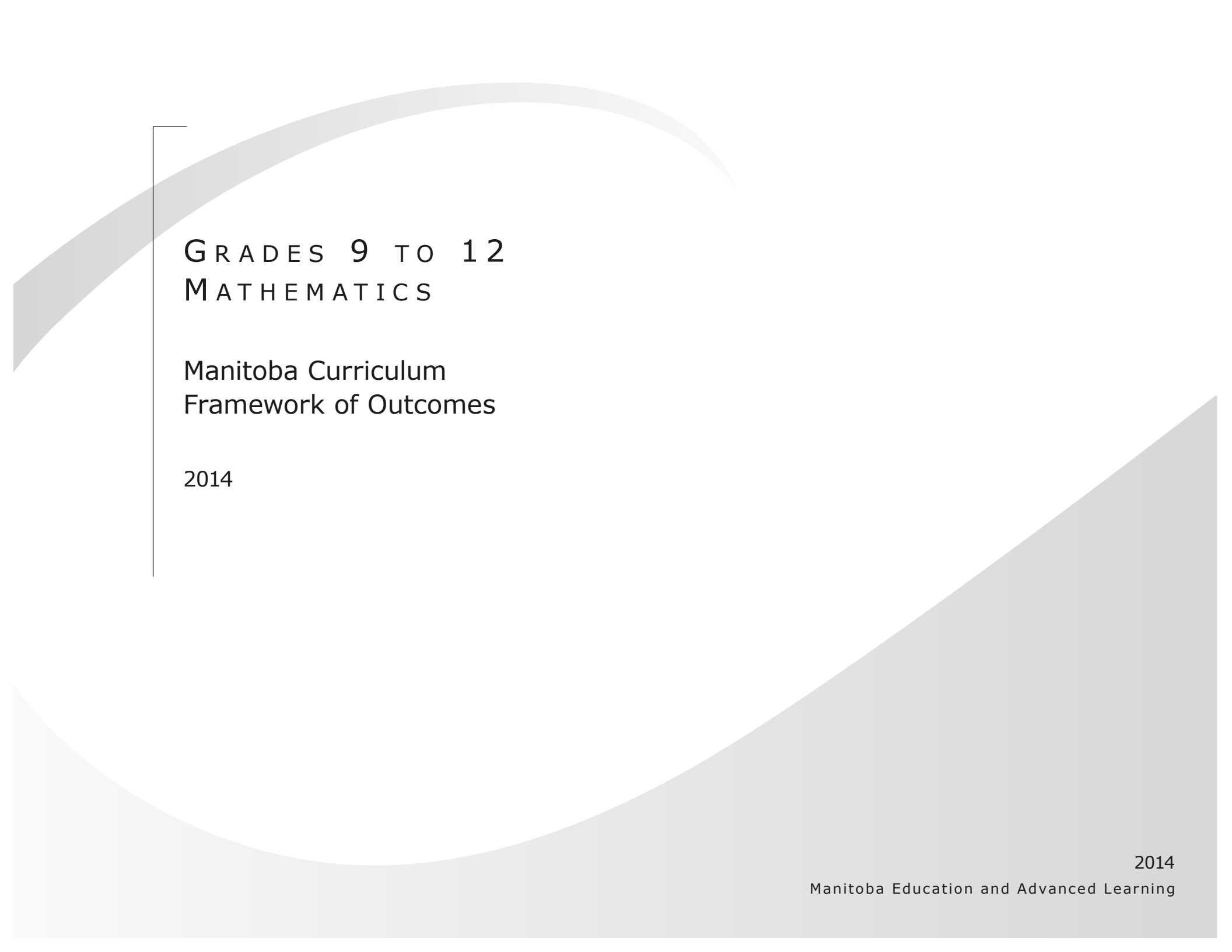




Grades 9 to 12 Mathematics

Manitoba Curriculum
Framework of Outcomes

2014



GRADES 9 TO 12 MATHEMATICS

Manitoba Curriculum
Framework of Outcomes

2014

2014

Manitoba Education and Advanced Learning

Manitoba Education and Advanced Learning Cataloguing in Publication Data

Grades 9 to 12 mathematics [electronic resource] : Manitoba curriculum framework of outcomes. 2014

Based on 'The common curriculum framework for K-9 mathematics' (2006) and 'The common curriculum framework for Grades 10-12 mathematics' (2008)/developed by the Western and Northern Canadian Protocol.

Includes bibliographical references.

ISBN: 978-0-7711-5804-9

1. Mathematics—Study and teaching (Secondary).
 2. Mathematics—Study and teaching (Secondary)—Manitoba.
 3. Mathematics—Manitoba—Curricula.
 - I. Manitoba. Manitoba Education and Advanced Learning.
 - II. Title: The common curriculum framework for K-9 mathematics.
 - III. Title: The common curriculum framework for Grades 10-12 mathematics.
- 510.71

Copyright © 2014, the Government of Manitoba, represented by the Minister of Education and Advanced Learning.

Manitoba Education and Advanced Learning
School Programs Division
Winnipeg, Manitoba, Canada

Every effort has been made to acknowledge original sources and to comply with copyright law. If cases are identified where this has not been done, please notify Manitoba Education and Advanced Learning. Sincere thanks to the authors, artists, and publishers who allowed their original material to be used.

Permission is hereby given to reproduce this document for non-profit educational purposes provided the source is cited.

Any websites referenced in this document are subject to change.

Print copies of this resource can be purchased from the Manitoba Text Book Bureau (stock number 80627). Order online at <www.mtbb.mb.ca>.

This resource is also available on the Manitoba Education and Advanced Learning website at <www.edu.gov.mb.ca/k12/cur/math/frameworks.html>.

Websites are subject to change without notice.

Disponible en français.

Available in alternate formats upon request.

CONTENTS

Acknowledgements	v
Background	1
Introduction	3
Purpose of the Document	3
Beliefs about Students and Mathematics Learning	3
First Nations, Métis, and Inuit Perspectives	4
Affective Domain	4
Goals for Students	5
Conceptual Framework for Grades 9 to 12 Mathematics	7
Mathematical Processes	8
Nature of Mathematics	12
Pathways and Topics	15
Outcomes and Achievement Indicators	17
Summary	18
Instructional Focus	19

General and Specific Learning Outcomes by Strand	21
Applied Mathematics	23
Number	23
Patterns and Relations	25
Shape and Space	32
Statistics and Probability	35
Essential Mathematics	38
Number	38
Patterns and Relations	40
Shape and Space	44
Statistics and Probability	50
Pre-Calculus Mathematics	53
Number	53
Patterns and Relations	55
Shape and Space	64
Statistics and Probability	66

General and Specific Learning Outcomes with Achievement Indicators by Course	69
Grade 9 Mathematics (10F)	69
Grade 10 Essential Mathematics (20S)	85
Grade 10 Essential Mathematics – Half Course I	87
Grade 10 Essential Mathematics – Half Course II	93
Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S)	99
Grade 11 Applied Mathematics (30S)	111
Grade 11 Essential Mathematics (30S)	123
Grade 11 Essential Mathematics – Half Course III	125
Grade 11 Essential Mathematics – Half Course IV	131
Grade 11 Pre-Calculus Mathematics (30S)	139
Grade 12 Applied Mathematics (40S)	155
Grade 12 Essential Mathematics (40S)	169
Grade 12 Essential Mathematics – Half Course V	171
Grade 12 Essential Mathematics – Half Course VI	176
Grade 12 Pre-Calculus Mathematics (40S)	181
References	199

ACKNOWLEDGEMENTS

The Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes is a revision of the Western and Northern Canadian Protocol (WNCP) *The Common Curriculum Framework for K–9 Mathematics* and *The Common Curriculum Framework for Grades 10–12 Mathematics*, developed through the cooperative efforts of the four western provinces and three territories. Manitoba Education and Advanced Learning would like to acknowledge the following:

**Grade 9 Mathematics
Development Team
(until June 2009)**

Ross Acree	Rolling River School Division
Don Bradford	St. James-Assiniboia School Division
Harry Hodges	Park West School Division
Carol Larson	Swan Valley School Division
Kelly Scallion	Mystery Lake School District
Ethel Stevenson	Peguis Central School
Shona Treleaven	Winnipeg School Division (retired)
Jerrold Wiebe	Pembina Trails School Division

**Grades 10 to 12 Applied
Mathematics Development Team
(until June 2009)**

Cam Bennet	Mountain View School Division
Troy Brister	Beautiful Plains School Division
Julie Cordova	St. James-Assiniboia School Division
Steven Erickson	Winnipeg School Division
Ross Forfar	Evergreen School Division
Leanne Funk Brown	Louis Riel School Division

**Grades 10 to 12 Essential
Mathematics Development Team
(until June 2009)**

April Popple	Southwest Horizon School Division
Brent Richards	Brandon School Division
Kim Sklepowich	Sunrise School Division
Suzanne Dunwoody	Seven Oaks School Division
Alice Harms	Mennonite Collegiate Institute
Ted Jeninga	Calvin Christian Collegiate
Curt Lothar	Winnipeg School Division
Carrie Malanchuk	Sunrise School Division
Kennet Mann	Winnipeg School Division
Joe Oczerklewicz	St. James-Assiniboia School Division
Edith Steeves	Mystery Lake School District

**Grades 10 to 12 Pre-Calculus
Mathematics Development Team
(until June 2009)**

Dawn Birch	Sunrise School Division
Sherri Burroughs	St. John's Ravenscourt
Melani Decelles	Winnipeg School Division
Ian Donnelly	River East Transcona School Division
Michael Dyck	Garden Valley School Division
Brian Hutton	Frontier School Division
Lyn Jannuska	Brandon School Division
Ryan Maksymchuk	Swan Valley School Division
Scott Smith	Whiteshell School Division

French Committee (until 2009)	Julie Bacon-Papineau	Seine River School Division
	Diane Lavergne	River East Transcona School Division
	Nicole Saint-Vincent	Division scolaire franco-manitobaine
	Gilles Vermette	Louis Riel School Division
	Richard Verrier	Division scolaire franco-manitobaine
School Programs Division, Manitoba Education and Advanced Learning	Nicole Allain Fox Consultant	Curriculum, Development and Implementation Bureau de l'éducation française Division
	Carole Bilyk Project Leader	Development Unit Instruction, Curriculum and Assessment Branch
	Louise Boissonneault Publications Editor	Document Production Services Unit Educational Resources Branch
	Lee-Ila Bothe Coordinator (until March 2010)	Document Production Services Unit Educational Resources Branch
	Chris Carman Consultant (until June 2014)	Assessment Unit Instruction, Curriculum and Assessment Branch
	Ian Donnelly Consultant (since February 2012)	Development Unit Instruction, Curriculum and Assessment Branch
	Philippe Leclercq Consultant	Curriculum, Development and Implementation Bureau de l'éducation française Division
	Gilbert Le Néal Consultant (until June 2008)	Curriculum, Development and Implementation Bureau de l'éducation française Division
	King Luu Consultant (since June 2013)	Assessment Unit Instruction, Curriculum and Assessment Branch
	Grant Moore Publications Editor	Document Production Services Unit Educational Resources Branch

Cyril Parent
Desktop Publisher

Allison Potter
Consultant (until June 2013)

Barbara Riou
Consultant (until December 2012)

Youyi Sun
Consultant (since May 2013)

Document Production Services Unit
Educational Resources Branch

Assessment Unit
Instruction, Curriculum and Assessment Branch

Assessment Unit
Instruction, Curriculum and Assessment Branch

Assessment Unit
Instruction, Curriculum and Assessment Branch

BACKGROUND

In December 1993, the Ministers of Education from Alberta, British Columbia, Manitoba, Northwest Territories, Saskatchewan, and Yukon Territory signed the Western Canadian Protocol (WCP) for Collaboration in

WNCP jurisdictions:

Alberta
British Columbia
Manitoba
Northwest Territories
Nunavut
Saskatchewan
Yukon Territory

Basic Education (Kindergarten to Grade 12). In February 2000, following the addition of Nunavut, the protocol was renamed the Western and Northern Canadian Protocol (WNCP) for Collaboration in Education (Kindergarten to Grade 12).

In 2005, the Ministers of Education from all the WNCP jurisdictions unanimously concurred with

the rationale of the original partnership because of the importance placed on

- common educational goals
- the ability to collaborate to achieve common goals
- high standards in education
- planning an array of educational opportunities
- removing obstacles to accessibility for individual learners
- optimum use of limited educational resources

The original WCP common curriculum framework for mathematics was published in two documents: Kindergarten to Grade 9 in 1995 and Grades 10 to 12 in 1996.

The Common Curriculum Framework for K–9 Mathematics: Western and Northern Canadian Protocol (May 2006) and *The Common Curriculum Framework for Grades 10–12 Mathematics: Western and Northern Canadian Protocol* (January 2008) were developed by the seven ministries of education in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The *Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* is based on both WNCP common curriculum framework documents.

The framework identifies beliefs about mathematics learning and teaching, general and specific learning outcomes, and achievement indicators agreed upon by the seven jurisdictions. Each of the provinces and territories will determine when and how the framework will be implemented within its own jurisdiction.

INTRODUCTION

Purpose of the Document

This document provides sets of learning outcomes to be used as a common base for defining mathematics

The framework communicates high expectations for students' mathematical learnings.

curriculum expectations that will be mandated in Grades 9, 10, 11, and 12. This common base should result in consistent student outcomes in mathematics across the WNCP jurisdictions and enable easier transfer for students moving from one jurisdiction to another.

This document is intended to

clearly communicate high expectations for students' mathematical learnings in Grades 9, 10, 11, and 12 to all education partners across the jurisdictions, and to facilitate the development of common learning resources.

Beliefs about Students and Mathematics Learning

Mathematical understanding is fostered when students build on their own experiences and prior knowledge.

Students are curious, active learners with individual interests, abilities, needs, and career goals. They come to school with varying knowledge, life experiences, expectations, and backgrounds. A key component in developing mathematical literacy in students is

making connections to these backgrounds, experiences, goals, and aspirations.

Students construct their understanding of mathematics by developing meaning based on a variety of learning experiences.

This meaning is best developed when learners encounter mathematical experiences that proceed from simple to complex and from the concrete to the abstract. The use of manipulatives, visuals, and a variety of pedagogical and assessment approaches can address the diversity of learning styles and developmental stages of students. At all levels of understanding, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions also provide essential links among concrete, pictorial, and symbolic representations of mathematics.

Students need frequent opportunities to develop and reinforce their conceptual understanding, procedural thinking, and problem-solving abilities. By addressing these three interrelated components, students will strengthen their ability to apply mathematical learning to their daily lives.

"Conceptual understanding is defined as comprehending mathematical concepts, relations, and operations to build new knowledge." (Kilpatrick, Swafford, and Findell 5)

Procedural thinking involves "carrying out procedures flexibly, accurately, efficiently, and appropriately."

Problem solving involves "engaging in understanding and resolving problem situations where a method or solution is not immediately obvious." (OECD 12)

The learning environment should value, respect, and address all students' experiences and ways of thinking, so that students are comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore mathematics through solving problems in order to continue developing personal strategies and mathematical literacy. It is important to realize that it is acceptable to solve problems in different ways and that solutions may vary depending upon how the problem is understood.

Assessment *for* learning, assessment *as* learning, and assessment *of* learning are all critical to helping students learn mathematics. A variety of evidence and a variety of assessment approaches should be used in the mathematics classroom.

First Nations, Métis, and Inuit Perspectives

First Nations, Métis, and Inuit students in Manitoba come from diverse geographic areas and have varied cultural and linguistic backgrounds. Students attend schools in a

Teachers need to understand the diversity of students' cultures and experiences.

variety of settings, including urban, rural, and isolated communities. Teachers need to recognize and understand the diversity of cultures within schools and the diverse experiences of students.

First Nations, Métis, and Inuit students often have a whole-world view of the environment; as a result, many of these students live and learn best in a holistic way. This means that students look for connections in learning and learn mathematics

best when it is contextualized and not taught as discrete content.

Many First Nations, Métis, and Inuit students come from cultural environments where learning takes place through active participation. Traditionally, little emphasis was placed upon the written word. Oral communication, along with practical applications and experiences, is important to student learning and understanding.

A variety of teaching and assessment strategies are required to build upon the diverse knowledge, cultures, communication styles, skills, attitudes, experiences, and learning styles of students.

The strategies used must go beyond the incidental inclusion of topics and objects unique to a culture or region and strive to achieve higher levels of multicultural education (Banks and Banks, 1993).

Affective Domain

A positive attitude is an important aspect of the affective domain that has a profound effect on learning.

Environments that create a sense of belonging, support risk taking, and provide opportunities for success help students to develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, to participate willingly in

To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

classroom activities, to persist in challenging situations, and to engage in reflective practices.

Teachers, students, and parents need to recognize the relationship between the affective and cognitive domains

Curiosity about mathematics is fostered when students are actively engaged in their environment.

and to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must be taught to set achievable goals and assess themselves as they work toward these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Goals for Students

The main goals of mathematics education are to prepare students to

Mathematics education must prepare students to use mathematics to think critically about the world.

- communicate and reason mathematically
- use mathematics confidently, accurately, and efficiently to solve problems
- appreciate and value mathematics
- make connections between

mathematical knowledge and skills and their applications

- commit themselves to lifelong learning
- become mathematically literate citizens, using mathematics to contribute to society and to think critically about the world

Students who have met these goals

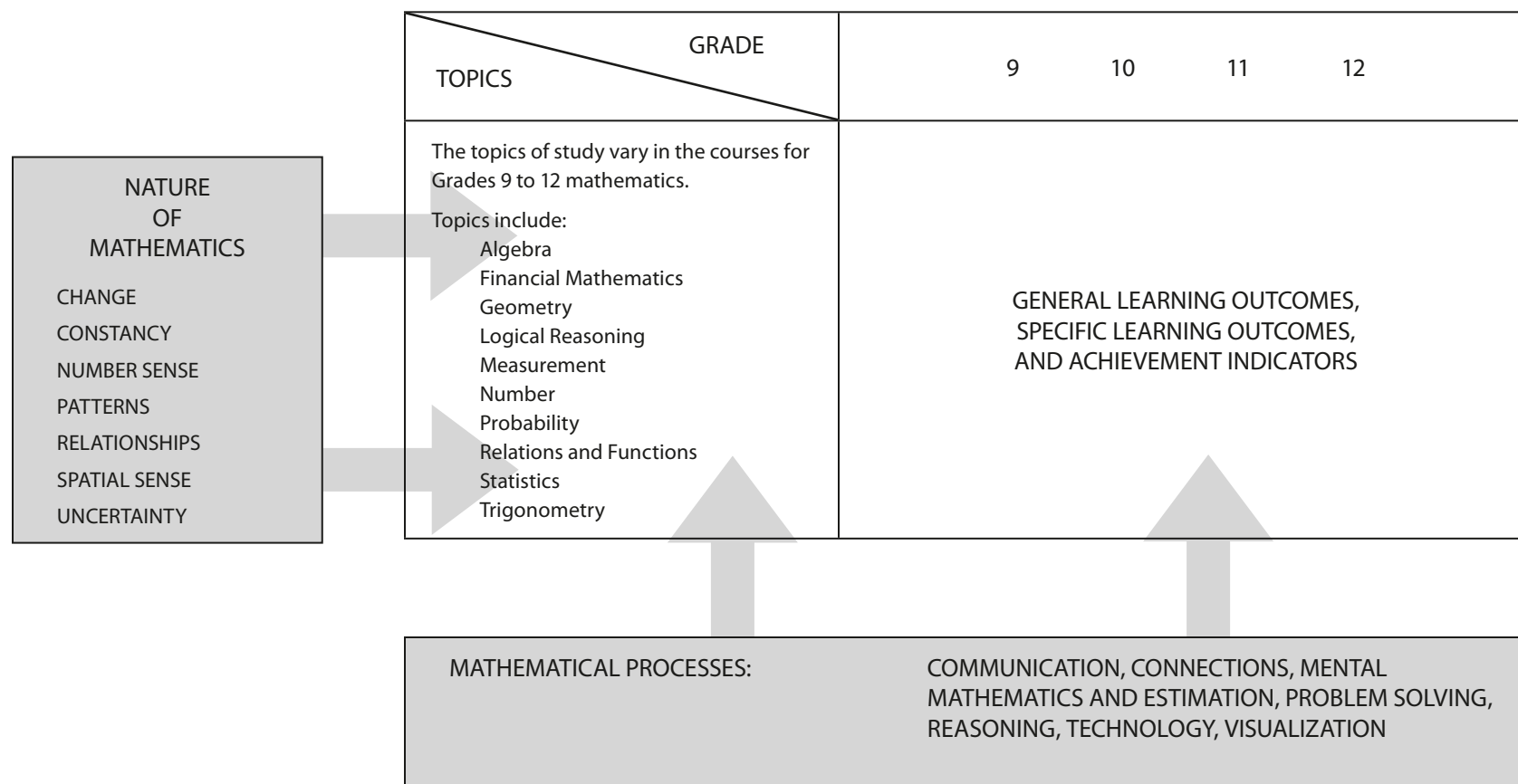
- gain an understanding and appreciation of the role of mathematics in society
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical problem solving
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

In order to assist students in attaining these goals, teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding through

- taking risks
- thinking and reflecting independently
- sharing and communicating mathematical understanding
- solving problems in individual and group projects
- pursuing greater understanding of mathematics
- appreciating the value of mathematics throughout history

CONCEPTUAL FRAMEWORK FOR GRADES 9 TO 12 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



Mathematical Processes

The seven mathematical processes are critical aspects of learning, doing, and understanding mathematics. Students must encounter these processes regularly in a mathematics program in order to achieve the goals of mathematics education.

The common curriculum framework incorporates the following interrelated mathematical processes. It is intended that they permeate the teaching and learning of mathematics.

Students are expected to

- Communication [C]
- Connections [CN]
- Mental Mathematics and Estimation [ME]
- Problem Solving [PS]
- Reasoning [R]
- Technology [T]
- Visualization [V]

- use *communication* in order to learn and express their understanding
- make *connections* among mathematical ideas, other concepts in mathematics, everyday experiences, and other disciplines
- demonstrate fluency with *mental mathematics and estimation*
- develop and apply new mathematical knowledge through *problem solving*
- develop mathematical *reasoning*
- select and use *technology* as a tool for learning and solving problems
- develop *visualization* skills to assist in processing information, making connections, and solving problems

All seven processes should be used in the teaching and learning of mathematics. Each specific learning outcome includes a list of relevant mathematical processes. All seven processes should be incorporated into learning experiences but the identified processes are to be used as a primary focus of instruction and assessment.

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links among their own language and ideas, the language and ideas of others, and the formal language and symbols of mathematics.

Students must be able to communicate mathematical ideas in a variety of ways and contexts.

Communication is important in clarifying, reinforcing, and modifying ideas, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication can play a significant role in helping students make connections among concrete, pictorial, graphical, symbolic, verbal, written, and mental representations of mathematical ideas. Explanation of ideas should use the various representations as appropriate.

Emerging technologies enable students to engage in communication beyond the traditional classroom to gather data and share mathematical ideas.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant, and integrated.

Through connections, students begin to view mathematics as useful and relevant.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. “Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding...Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching” (Caine and Caine 5).

Mental Mathematics and Estimation [ME]

Mental mathematics and estimation is a combination of cognitive strategies that enhance flexible thinking and

number sense. It involves using strategies to perform mental calculations.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility in reasoning and calculating.

“Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math” (National Council of Teachers of Mathematics, May 2005).

Mental mathematics and estimation are fundamental components of number sense.

Students proficient with mental mathematics “become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein 442).

Mental mathematics “provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers” (Hope v).

Estimation is used for determining approximate values or quantities, usually by referring to benchmarks or referents, or for determining the reasonableness of calculated values. Estimation is also used to make mathematical judgments and to develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to learn which strategy to use and how to use it.

To help students become efficient with computational fluency, students need to develop mental math skills and recall math facts automatically. Learning math facts is a developmental process where the focus of instruction is on thinking and building number relationships. Facts become automatic for students through repeated exposure and practice. When a student recalls facts, the answer should be produced without resorting to inefficient means, such as counting. When facts are automatic, students are no longer using strategies to retrieve them from memory.

Problem Solving [PS]

“Problem solving is an integral part of all mathematics learning” (NCTM, *Problem Solving*). Learning through problem solving should be the focus of mathematics at all grade levels. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts. Problem solving is to be employed throughout all of mathematics and should be embedded throughout all the topics.

When students encounter new situations and respond to questions of the type, *How would you...?* or *How could you...?*, the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing, and trying different strategies.

Learning through problem solving should be the focus of mathematics at all grade levels.

In order for an activity to be based on problem solving, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but

practice. Students should not know the answer immediately. A true problem requires students to use prior knowledge in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement. Students will be engaged if the problems relate to their lives, cultures, interests, families, or current events.

Both conceptual understanding and student engagement are fundamental in moulding students’ willingness to persevere in future problem-solving tasks.

Problems are not just simple computations embedded in a story, nor are they contrived. They are tasks that are rich and open-ended, so there may be more than one way of arriving at a solution or there may be multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill, or understanding. Problem solving can vary from being an individual activity to a class (or beyond) undertaking.

In a mathematics class, there are two distinct types of problem solving: solving contextual problems outside of mathematics and solving mathematical problems. Finding the maximum profit given manufacturing constraints is an example of a contextual problem, while seeking and developing a general formula to solve a quadratic equation is an example of a mathematical problem.

Problem solving can also be considered in terms of engaging students in both inductive and deductive reasoning strategies. As students make sense of the problem, they will be creating conjectures and looking for patterns that they may be able to generalize. This part of the problem-solving process often involves inductive reasoning. As students use approaches to solving the

problem they often move into mathematical reasoning that is deductive in nature. It is crucial that students be encouraged to engage in both types of reasoning and be given the opportunity to consider the approaches and strategies used by others in solving similar problems.

Problem solving is a powerful teaching tool that fosters multiple creative and innovative solutions. Creating an environment where students openly look for, and engage in, finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk-takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop

Mathematical reasoning helps students think logically and make sense of mathematics.

confidence in their abilities to reason and justify their mathematical thinking. Questions that challenge students to think, analyze, and synthesize help them develop an understanding of mathematics. All students need to be challenged to answer questions

such as *“Why do you believe that’s true/correct?”* or *“What would happen if...?”*.

Mathematical experiences provide opportunities for students to engage in inductive and deductive reasoning. Students use inductive reasoning when they explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Students use deductive reasoning when they reach new conclusions

based upon the application of what is already known or assumed to be true. The thinking skills developed by focusing on reasoning can be used in daily life in a wide variety of contexts and disciplines.

When explaining ideas, students should be encouraged to use concrete, pictorial, symbolic, graphical, verbal, and written representations of their mathematical ideas.

Technology [T]

Technology can be used effectively to contribute to and support the learning of a wide range of mathematical learning outcomes. Technology enables students to explore and create patterns, examine relationships, test conjectures, and solve problems. Students in Grades 9 to 12 are expected to have consistent access to technology for their mathematics courses.

Technology has the potential to enhance the teaching and learning of mathematics. It can be used to

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- generate and test inductive conjectures
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems

Technology contributes to the learning of a wide range of mathematical outcomes, and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

- increase the focus on conceptual understanding by decreasing the time spent on repetitive procedures
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- model situations
- develop number and spatial sense
- create geometric figures

Technology contributes to a learning environment in which the curiosity of students can lead to rich mathematical discoveries at all grade levels.

Students need to know when it is appropriate to use technology such as a calculator and when to apply their mental computation, reasoning, and estimation skills to predict and check answers. The use of technology can enhance, although it should not replace, conceptual understanding, procedural thinking, and problem solving.

Visualization [V]

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world” (Armstrong 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and spatial reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure and when to estimate and involves knowledge of several estimation strategies (Shaw and Cliatt 150).

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations. It is through visualization that abstract concepts can be understood concretely by the student. Visualization is a foundation to the development of abstract understanding, confidence, and fluency.

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Nature of Mathematics

Mathematics is one way of understanding, interpreting, and describing our world. There are a number of characteristics that define the nature of mathematics, including change, constancy, number sense, patterns, relationships, spatial sense, and uncertainty.

- Change
- Constancy
- Number Sense
- Patterns
- Relationships
- Spatial Sense
- Uncertainty

Change

It is important for students to understand that mathematics is dynamic and not static. As a result,

Change is an integral part of mathematics and the learning of mathematics.

recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To

make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12... can be described as

- skip-counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain (Steen 184)

Constancy

Many important properties in mathematics do not change when conditions change. Examples of constancy include

- the conservation of equality in solving equations
- the sum of the interior angles of any triangle
- theoretical probability of an event

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems such as those involving constant rates of change, lines with constant slope, or direct variation situations.

Some problems in mathematics require students to focus on properties that remain constant.

Number Sense

Number sense, which can be thought of as deep understanding and flexibility with numbers, is the most important foundation of numeracy (British Columbia Ministry of Education 146). Continuing to foster number sense is fundamental to growth of mathematical understanding.

Number sense is an awareness and understanding of what numbers are.

Number sense is an awareness and understanding of what numbers are, their relationships, their magnitude, and the relative effect of operating on numbers, including the use of mental mathematics and estimation (Fennell and Landis 187).

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns.

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns.

Patterns exist in all of the mathematical topics, and it is through the study of patterns that students can make strong connections between concepts in the same and different topics. Working with patterns also enables students to make connections beyond mathematics. The ability to

analyze patterns contributes to how students understand their environment.

Patterns may be represented in concrete, visual, auditory, or symbolic form. Students should develop fluency in moving from one representation to another.

Students need to learn to recognize, extend, create, and apply mathematical patterns. This understanding of patterns allows students to make predictions and justify their reasoning when solving problems.

Learning to work with patterns helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics.

Relationships

Mathematics is used to describe and explain relationships. Within the study of mathematics, students look for relationships among numbers, sets, shapes, objects, variables, and concepts. The search for possible relationships involves collecting and analyzing data, analyzing patterns, and describing possible relationships visually, symbolically, orally, or in written form. Technology should be used to aid in the search for relationships.

Mathematics is used to describe and explain relationships.

Spatial Sense

Spatial sense involves the representation and manipulation of 3-D objects and 2-D shapes. It enables students to reason and interpret among 3-D and 2-D representations.

Spatial sense offers a way to interpret and reflect on the physical environment.

Spatial sense is developed through a variety of experiences with visual and concrete models. Some of these experiences should involve the use of technology. These experiences offer a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions.

Spatial sense is also critical in students' understanding of the relationship between the equations and graphs of functions and, ultimately, in understanding how both equations and graphs can be used to represent physical situations. Graphing calculators or graphing software can aid students in developing this understanding.

Uncertainty

In mathematics, interpretations of data and the predictions made from data inherently lack certainty.

Uncertainty is an inherent part of making predictions.

Events and experiments generate statistical data that can be used to make predictions. It is important that students recognize that these predictions (interpolations and extrapolations) are based upon

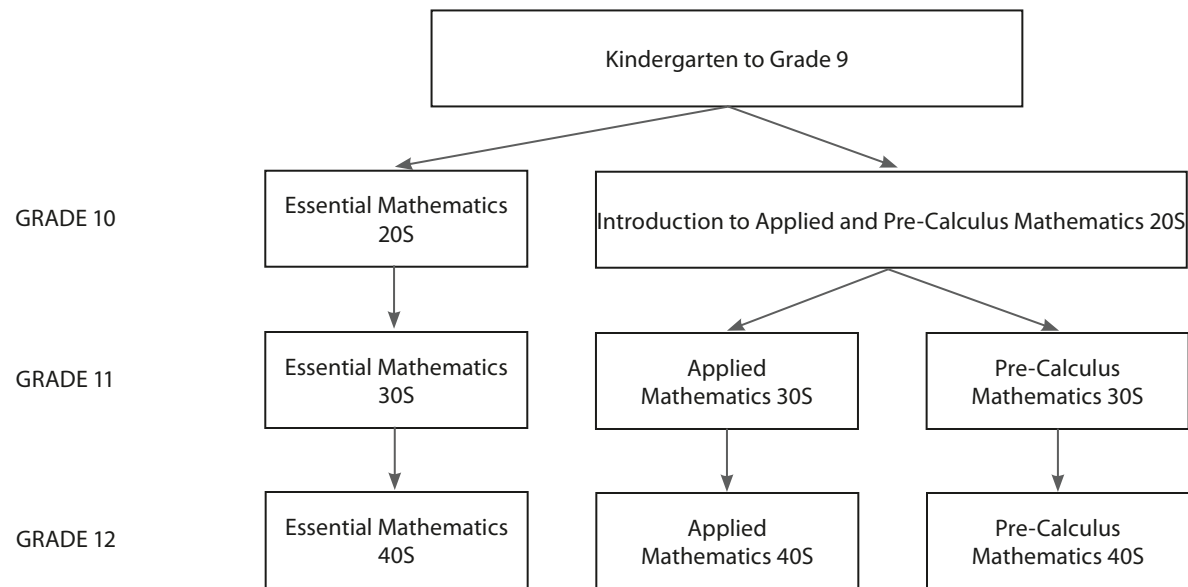
patterns that have a degree of uncertainty. The quality of an interpretation or conclusion is directly related to the quality of the data it is based upon. An awareness of uncertainty provides students with an understanding of why and how to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately. This language must be used effectively and correctly to convey valuable messages.

Pathways and Topics

Unlike *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes, Grades 9 to 12 Mathematics: Manitoba Curriculum Framework of Outcomes* includes topics rather than strands. Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings.

There is one course available for students in Grade 9. In Grade 10, students may choose between two courses or may choose to take both courses. In Grades 11 and 12, students have three choices for courses and may take one or multiple courses.



Goals of Pathways

The goals of all pathways are to provide prerequisite attitudes, knowledge, skills, and understandings for specific post-secondary programs or direct entry into the workforce. All three pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. When choosing a pathway, students should consider their interests, both current and future. Students, parents, and educators are encouraged to research the admission requirements for post-secondary programs of study, as they vary by institution and by year.

Design of Pathways

Each pathway is designed to provide students with the mathematical understandings, rigour, and critical-thinking skills that have been identified for specific post-secondary programs of study and for direct entry into the workforce.

The content of each pathway has been based on the *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings* and on consultations with mathematics teachers.

Applied Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require the study of theoretical calculus. Topics include financial mathematics, geometry, logical reasoning, measurement, number, relations and functions, and statistics and probability.

Essential Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that do not require further study in advanced mathematics. Topics include algebra, geometry, measurement, number, statistics and probability, and financial mathematics.

Pre-Calculus Mathematics

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, permutations, combinations and binomial theorem, relations and functions, and trigonometry.

Learning Outcomes and Achievement Indicators

The common curriculum framework is stated in terms of general learning outcomes, specific learning outcomes, and achievement indicators.

General learning outcomes are overarching statements about what students are expected to learn in each course. They remain consistent throughout several years of schooling.

Specific learning outcomes are statements that identify the specific knowledge, skills, and understandings that students are required to attain by the end of a given course. Some learning outcomes will be revisited several times during a course to allow for connections to be made to other learning outcomes in the course.

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific learning outcome. The range of samples provided is meant to reflect the depth, breadth, and expectations of the specific learning outcome. While they provide some examples of student achievement, they are not meant to reflect the sole indicators of success. They are not presented in any particular order and need not be explicitly addressed in the classroom. However, students need to understand the outcomes at least to the depth indicated by the indicators. Therefore, the achievement indicators are sufficient as a basis for instructional design and assessment, and will form the basis for provincial assessment as appropriate.

- In the specific learning outcomes and achievement indicators, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome.
- In the specific learning outcomes and achievement indicators, the words *such as* indicate that the ensuing items are provided for clarification and are not requirements that must be addressed to fully meet the learning outcome.
- The word *and* used in a learning outcome indicates that both ideas must be addressed to fully meet the learning outcome, although not necessarily at the same time or in the same question.
- The word *and* used in an achievement indicator implies that both ideas should be addressed at the same time or in the same question.
- The word *or* used in an achievement indicator implies that both ideas should be addressed but not at the same time or in the same question.
- The word *concretely* implies students would use physical materials that they manipulate.
- The word *pictorially* implies students would create pictures of concrete materials.
- The word *symbolically* implies students would use variables, numbers, and mathematical signs.

Summary

The Conceptual Framework for Grades 9 to 12 Mathematics describes the nature of mathematics, the mathematical processes, the pathways and topics, and the role of learning outcomes and achievement indicators in Grades 9 to 12 mathematics. Activities that take place in the mathematics classroom should be based on a problem-solving approach that incorporates the mathematical processes and leads students to an understanding of the nature of mathematics.

INSTRUCTIONAL FOCUS

Learning in mathematics is richer and more engaging when instruction and assessment develop a direct relationship between conceptual and procedural understanding. Students should be engaged in making connections among concepts both within and across topics to make mathematical learning experiences meaningful.

Teachers should consider the following points when planning for instruction and assessment.

- Students in Manitoba bring a diversity of learning styles and cultural backgrounds to the classroom and they may be at varying developmental stages. Methods of instruction should be based on the learning styles and abilities of the students.
- Learning outcomes need to be organized into units of study. Each course suggests at least one possible order but teachers need to decide which order works best in their unique context.
- The mathematical processes that are identified with the learning outcome are intended to help teachers select effective pedagogical approaches for the teaching and learning of the outcome.
- All seven mathematical processes must be integrated throughout teaching and learning approaches, and should support the intent of the learning outcomes.
- Wherever possible, meaningful contexts should be used in examples, problems, and projects.
- Instruction should flow from simple to complex and from concrete to abstract.
- Students are expected to have consistent access to technology for all mathematics courses.
- Routinely incorporating conceptual understanding, procedural thinking, and problem solving within instructional design will enable students to master the mathematical skills and concepts of the curriculum.
- Continuous professional learning related to exemplary practices supported by pedagogical research should inform instruction and assessment.
- The assessment plan for the course should be a balance of assessment *for* learning, assessment *as* learning, and assessment *of* learning.
- The assessment plan for the course should be a balance of multiple assessment tools including
 - assignments
 - journal entries
 - performance tasks
 - portfolios
 - projects
 - quizzes
 - tests
- Positive, timely, descriptive feedback should be used to allow students to deepen their understanding of the mathematical concepts and processes.
- Teachers should organize reports about learning in mathematics by learning outcome rather than by assessment tool to show a profile of student strengths and challenges. Grading should reflect achievement of the learning outcomes, separate from effort, participation, or attitude.

“Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways—individually, cooperatively, independently, with teacher direction, through hands-on experience, through examples followed by practice. In addition, mathematics requires students to learn concepts and procedures, acquire skills, and learn and apply mathematical processes. These different areas of learning may involve different teaching and learning strategies. It is assumed, therefore, that the strategies teachers employ will vary according to both the object of the learning and the needs of the students” (Ontario 24).

GENERAL AND SPECIFIC LEARNING OUTCOMES BY STRAND

This section presents the general and specific learning outcomes for each of Applied Mathematics, Essential Mathematics, and Pre-Calculus Mathematics, organized by the strands from the *Kindergarten to Grade 8 Mathematics Manitoba Curriculum Framework of Outcomes*. Each chart includes the outcomes from Grades 8 to 12. For Applied Mathematics and Pre-Calculus Mathematics, the Grade 10 outcomes are identical and are from the Introduction to Applied and Pre-Calculus Mathematics 20S course.

The code for the Grades 10 to 12 learning outcomes is as follows: Grade, Course, Topic, Learning Outcome Number

Examples:

1. A code of 11A.L.1 is Grade 11 Applied Mathematics; Logical Reasoning topic, the first learning outcome of that topic.
2. A code of 10E2.C.1 is Grade 10 Essential Mathematics, Half Course II; Consumer Decisions topic, the first learning outcome of that topic.

Grade	Course	Topics
10	Essential Mathematics, Half Course I	Analysis of Games and Numbers, Personal Finance, Measurement, 2-D Geometry
10	Essential Mathematics, Half Course II	Analysis of Games and Numbers, Trigonometry, Consumer Decisions, Transformations, Angle Construction
10	Introduction to Applied and Pre-Calculus Mathematics	Measurement, Algebra and Number, Relations and Functions
11	Applied Mathematics	Measurement, Geometry, Logical Reasoning Statistics, Relations and Functions, Mathematics Research Project
11	Essential Mathematics, Half Course III	Analysis of Games and Numbers, Interest and Credit, 3-D Geometry, Statistics
11	Essential Mathematics, Half Course IV	Analysis of Games and Numbers, Managing Money, Relations and Patterns, Trigonometry, Design Modelling
11	Pre-Calculus Mathematics	Algebra and Number, Trigonometry, Relations and Functions
12	Applied Mathematics	Financial Mathematics, Logical Reasoning, Probability, Relations and Functions, Mathematics Research Project, Design and Measurement
12	Essential Mathematics, Half Course V	Analysis of Games and Numbers, Vehicle Finance, Statistics, Precision Measurement, Career Life
12	Essential Mathematics, Half Course VI	Analysis of Games and Numbers, Home Finance, Geometry and Trigonometry, Business Finance, Probability
12	Pre-Calculus Mathematics	Trigonometry, Permutations, Combinations and Binomial Theorem, Relations and Functions

General and Specific Learning Outcomes by Strand Applied Mathematics

Number

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Develop number sense.	General Learning Outcome Develop number sense.			
Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes		Specific Learning Outcomes
8.N.1. Demonstrate an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers). [C, CN, R, V] 8.N.2. Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). [C, CN, ME, R, T] 8.N.3. Demonstrate an understanding of percents greater than or equal to 0%. [CN, PS, R, V] 8.N.4. Demonstrate an understanding of ratio and rate. [C, CN, V] 8.N.5. Solve problems that involve rates, ratios, and proportional reasoning. [C, CN, PS, R]	9.N.1. Demonstrate an understanding of powers with integral bases (excluding base 0) and whole-number exponents by ■ representing repeated multiplication using powers ■ using patterns to show that a power with an exponent of zero is equal to one ■ solving problems involving powers [C, CN, ME, PS, R] 9.N.2. Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole-number exponents. [C, CN, ME, PS, R, T]	10I.A.1. Demonstrate an understanding of factors of whole numbers by determining ■ prime factors ■ greatest common factor ■ least common multiple ■ square root ■ cube root [CN, ME, R] 10I.A.2. Demonstrate an understanding of irrational numbers by ■ representing, identifying, and simplifying irrational numbers ■ ordering irrational numbers [CN, ME, R, V] 10I.A.3. Demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R]		12A.FM.1. Solve problems that involve compound interest in financial decision making. [C, CN, PS, T, V] 12A.FM.2. Analyze costs and benefits of renting, leasing, and buying. [CN, PS, R, T] 12A.FM.3. Analyze an investment portfolio in terms of ■ interest rate ■ rate of return ■ total return [ME, PS, R, T] 12A.L.1. Analyze puzzles and games that involve numerical and logical reasoning, using problem-solving strategies. [CN, ME, PS, R, T]

Applied Mathematics

Number

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Develop number sense.	General Learning Outcome Develop number sense.			
Specific Learning Outcomes	Specific Learning Outcomes			
8.N.6. Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically. [C, CN, ME, PS]	9.N.3. Demonstrate an understanding of rational numbers by ■ comparing and ordering rational numbers ■ solving problems that involve arithmetic operations on rational numbers [C, CN, ME, PS, R, T, V]			
8.N.7. Demonstrate an understanding of multiplication and division of integers, concretely, pictorially, and symbolically. [C, CN, PS, R, V]	9.N.4. Explain and apply the order of operations, including exponents, with and without technology. [ME, PS, T]			
8.N.8. Solve problems involving positive rational numbers. [C, CN, ME, PS, R, T, V]	9.N.5. Determine the square root of positive rational numbers that are perfect squares. [C, CN, ME, PS, R, T]			
	9.N.6. Determine an approximate square root of positive rational numbers that are non-perfect squares. [C, CN, ME, PS, R, T]			

Applied Mathematics

Patterns and Relations (Patterns)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Use patterns to describe the world and solve problems.	General Learning Outcome Use patterns to describe the world and solve problems.			
Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes
8.PR.1. Graph and analyze two-variable linear equations. [C, ME, PS, R, T, V]	9.PR.1. Generalize a pattern arising from a problem-solving context using linear equations, and verify by substitution. [C, CN, PS, R, V] 9.PR.2. Graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems. [C, CN, ME, PS, R, T, V]	10I.R.1. Interpret and explain the relationships among data, graphs, and contexts. [C, CN, R, T, V]	11A.L.1. Analyze and prove conjectures, using inductive and deductive reasoning, to solve problems. [C, CN, PS, R, T]	12A.L.2. Solve problems that involve the application of set theory. [CN, PS, R, T, V] 12A.L.3. Solve problems that involve conditional statements. [C, CN, PS, R, T]

Applied Mathematics

Patterns and Relations (Variables and Equations)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Represent algebraic expressions in multiple ways.	General Learning Outcome Represent algebraic expressions in multiple ways.			
Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	
8.PR.2. Model and solve problems using linear equations of the form ■ $ax = b$ ■ $\frac{x}{a} = b, a \neq 0$ ■ $ax + b = c$ ■ $\frac{x}{a} + b = c, a \neq 0$ ■ $a(x + b) = c$ concretely, pictorially, and symbolically, where a, b, and c are integers. [C, CN, PS, V]	9.PR.3. Model and solve problems using linear equations of the form ■ $ax = b$ ■ $ax + b = c$ ■ $ax = b + cx$ ■ $a(x + b) = c$ ■ $ax + b = cx + d$ ■ $a(bx + c) = d(ex + f)$ ■ $\frac{a}{x} = b, x \neq 0$ where a, b, c, d, e, and f are rational numbers. [C, CN, ME, PS, V] 9.PR.4. Explain and illustrate strategies to solve single variable linear inequalities with rational number coefficients within a problem-solving context. [C, CN, ME, PS, R, V] 9.PR.5. Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). [C, CN, R, V]	10I.A.3. Demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R] 10I.A.4. Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials), concretely, pictorially, and symbolically. [C, CN, R, V] 10I.A.5. Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially, and symbolically. [C, CN, R, V]	11A.M.1. Solve problems that involve the application of rates. [CN, PS, R, T]	

Applied Mathematics

Patterns and Relations (Variables and Equations) *(continued)*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Represent algebraic expressions in multiple ways.				
Specific Learning Outcomes				
9.PR.6. Model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2). [C, CN, ME, PS, R, V]				
9.PR.7. Model, record, and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially, and symbolically. [C, CN, R, V]				

Applied Mathematics

Patterns and Relations (Relations and Functions)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
		Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes
		10I.R.1. Interpret and explain the relationships among data, graphs, and contexts. [C, CN, R, T, V]	11A.R.1. Model and solve problems that involve systems of linear inequalities in two variables. [CN, PS, T, V]	12A.R.1. Represent data, using polynomial functions (of degree ≤ 3), to solve problems. [C, CN, PS, T, V]
		10I.R.2. Demonstrate an understanding of relations and functions. [C, R, V]	11A.R.2. Demonstrate an understanding of the characteristics of quadratic functions, including	12A.R.2. Represent data, using exponential and logarithmic functions, to solve problems. [C, CN, PS, T, V]
		10I.R.3. Demonstrate an understanding of slope with respect to	■ vertex ■ intercepts ■ domain and range ■ axis of symmetry [CN, PS, T, V]	12A.R.3. Represent data, using sinusoidal functions, to solve problems. [C, CN, PS, T, V]
		■ rise and run ■ line segments and lines ■ rate of change ■ parallel lines ■ perpendicular lines [PS, R, V]		

Applied Mathematics

Patterns and Relations (Relations and Functions) *(continued)*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
Specific Learning Outcomes				
		10I.R.4. Describe and represent linear relations, using ■ words ■ ordered pairs ■ tables of values ■ graphs ■ equations [C, CN, R, V]		
		10I.R.5. Determine the characteristics of the graphs of linear relations, including the ■ intercepts ■ slope ■ domain ■ range [CN, PS, R, T, V]		

Applied Mathematics

Patterns and Relations (Relations and Functions) *(continued)*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
Specific Learning Outcomes				
		10I.R.6. Relate linear relations expressed in ■ slope-intercept form ($y = mx + b$) ■ general form ($Ax + By + c = 0$) ■ slope-point form ($y - y^1 = m(x - x^1)$) to their graphs. [C, CN, R, T, V] 10I.R.7. Determine the equation of a linear relation, given ■ a graph ■ a point and the slope ■ two points ■ a point and the equation of a parallel or perpendicular line ■ a scatterplot [C, CN, PS, R, T, V]		

Applied Mathematics

Patterns and Relations (Relations and Functions) *(continued)*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
		Specific Learning Outcomes		
		10I.R.8. Represent a linear function, using function notation. [CN, ME, V]		
		10I.R.9. Solve problems that involve systems of linear equations in two variables, graphically and algebraically. [CN, PS, R, T, V]		
		10I.R.10. Solve problems that involve the distance between two points and the midpoint of a line segment. [C, CN, PS, V]		

Applied Mathematics

Shape and Space (Measurement)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Use direct or indirect measurement to solve problems.	General Learning Outcome Use direct or indirect measurement to solve problems.			
Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes		Specific Learning Outcomes
8.SS.1. Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V] 8.SS.2. Draw and construct nets for 3-D objects. [C, CN, PS, V] 8.SS.3. Determine the surface area of ■ right rectangular prisms ■ right triangular prisms ■ right cylinders to solve problems. [C, CN, PS, R, V] 8.SS.4. Develop and apply formulas for determining the volume of right prisms and right cylinders. [C, CN, PS, R, V]	9.SS.1. Solve problems and justify the solution strategy using circle properties, including ■ the perpendicular from the centre of a circle to a chord bisects the chord ■ the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc ■ the inscribed angles subtended by the same arc are congruent ■ a tangent to a circle is perpendicular to the radius at the point of tangency [C, CN, PS, R, T, V]	10I.M.1. Solve problems that involve linear measurement, using ■ SI and imperial units of measure ■ estimation strategies ■ measurement strategies [ME, PS, V] 10I.M.2. Apply proportional reasoning to problems that involve conversions between SI and imperial units of measure. [C, ME, PS, T]		12A.D.1. Analyze objects, shapes, and processes to solve cost and design problems. [C, CN, ME, PS, R, T, V]

Applied Mathematics

Shape and Space (3-D Objects and 2-D Shapes)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.	General Learning Outcome Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.			
Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes
8.SS.5. Draw and interpret top, front, and side views of 3-D objects composed of right rectangular prisms. [C, CN, R, T, V]	9.SS.2. Determine the surface area of composite 3-D objects to solve problems. [C, CN, ME, PS, R, V] 9.SS.3. Demonstrate an understanding of similarity of polygons. [C, CN, PS, R, T, V]	10I.M.3. Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including ■ right cones ■ right cylinders ■ right prisms ■ right pyramids ■ spheres [CN, PS, R, T, V] 10I.M.4. Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles. [C, CN, PS, R, T, V]	11A.M.2. Solve problems that involve scale diagrams, using proportional reasoning. [CN, PS, R, T, V] 11A.M.3. Demonstrate an understanding of the relationships among scale factors, areas, surface areas, and volumes of similar 2-D shapes and 3-D objects. [C, CN, PS, R, T, V] 11A.G.1. Derive proofs that involve the properties of angles and triangles. [CN, R, T, V] 11A.G.2. Solve problems that involve the properties of angles and triangles. [CN, PS, T, V] 11A.G.3. Solve problems that involve the cosine law and the sine law, including the ambiguous case. [CN, PS, R, T]	12A.D.1. Analyze objects, shapes, and processes to solve cost and design problems. [C, CN, ME, PS, R, T, V]

Applied Mathematics

Shape and Space (Transformations)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Describe and analyze position and motion of objects and shapes.	General Learning Outcome Describe and analyze position and motion of objects and shapes.			
Specific Learning Outcomes	Specific Learning Outcomes		Specific Learning Outcomes	
8.SS.6. Demonstrate an understanding of tessellation by ■ explaining the properties of shapes that make tessellating possible ■ creating tessellations ■ identifying tessellations in the environment [C, CN, PS, T, V]	9.SS.4. Draw and interpret scale diagrams of 2-D shapes. [CN, R, T, V] 9.SS.5. Demonstrate an understanding of line and rotation symmetry. [C, CN, PS, V]		11A.L.2. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [CN, PS, R, T, V]	

Applied Mathematics

Statistics and Probability (Data Analysis)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Collect, display, and analyze data to solve problems.	General Learning Outcome Collect, display, and analyze data to solve problems.			
Specific Learning Outcomes	Specific Learning Outcomes		Specific Learning Outcomes	Specific Learning Outcomes
8.SP.1. Critique ways in which data are presented. [C, R, T, V]	9.SP.1. Describe the effect of ■ bias ■ use of language ■ ethics ■ cost ■ time and timing ■ privacy ■ cultural sensitivity on the collection of data. [C, CN, R, T] 9.SP.2. Select and defend the choice of using either a population or a sample of a population to answer a question. [C, CN, PS, R]		11A.S.1. Demonstrate an understanding of normal distribution, including ■ standard deviation ■ z-scores [CN, PS, T, V] 11A.S.2. Interpret statistical data, using ■ confidence intervals ■ confidence levels ■ margin of error [C, CN, R, T] 11A.RP.1. Research and give a presentation on a historical event or an area of interest that involves mathematics. [C, CN, ME, PS, R, T, V]	12A.RP.1. Research and give a presentation on a current event or an area of interest that involves mathematics. [C, CN, ME, PS, R, T, V]

Applied Mathematics

Statistics and Probability (Data Analysis) *(continued)*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Collect, display, and analyze data to solve problems.				
Specific Learning Outcomes				
9.SP. 3. Develop and implement a project plan for the collection, display, and analysis of data by ■ formulating a question for investigation ■ choosing a data collection method that includes social considerations ■ selecting a population or a sample ■ collecting the data ■ displaying the collected data in an appropriate manner ■ drawing conclusions to answer the question [C, PS, R, T, V]				

Applied Mathematics

Statistics and Probability (Chance and Uncertainty)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.	General Learning Outcome Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.			
Specific Learning Outcomes	Specific Learning Outcomes			Specific Learning Outcomes
8.SP.2. Solve problems involving the probability of independent events. [C, CN, PS, T]	9.SP.4. Demonstrate an understanding of the role of probability in society. [C, CN, R, T]			12A.P.1. Interpret and assess the validity of odds and probability statements. [C, CN, ME, T] 12A.P.2. Solve problems that involve the probability of mutually exclusive and non-mutually exclusive events. [CN, PS, R, T, V] 12A.P.3. Solve problems that involve the probability of independent and dependent events. [CN, PS, R, T] 12A.P.4. Solve problems that involve the fundamental counting principle. [PS, R, T, V] 12A.P.5. Solve problems that involve permutations. [ME, PS, R, T, V] 12A.P.6. Solve problems that involve combinations. [ME, PS, R, T, V]

General and Specific Learning Outcomes by Strand

Essential Mathematics

Number

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Develop number sense.	General Learning Outcome Develop number sense.			
Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes
8.N.1. Demonstrate an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers). [C, CN, R, V] 8.N.2. Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). [C, CN, ME, R, T] 8.N.3. Demonstrate an understanding of percents greater than or equal to 0%. [CN, PS, R, V] 8.N.4. Demonstrate an understanding of ratio and rate. [C, CN, V] 8.N.5. Solve problems that involve rates, ratios, and proportional reasoning. [C, CN, PS, R]	9.N.1. Demonstrate an understanding of powers with integral bases (excluding base 0) and whole-number exponents by ■ representing repeated multiplication using powers ■ using patterns to show that a power with an exponent of zero is equal to one ■ solving problems involving powers [C, CN, ME, PS, R] 9.N.2. Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole-number exponents. [C, CN, ME, PS, R, T]	10E2.C.1. Solve problems that involve unit pricing and currency exchange, using proportional reasoning. [CN, ME, PS, R] 10E1.P.1. Demonstrate an understanding of calculations for gross pay and net pay earned through income sources including ■ wages ■ salary ■ commissions ■ piecework [C, CN, R, T] 10E1.P.2. Solve problems that require the manipulation and application of formulas related to income. [C, CN, ME, PS, R]	11E3.A.1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R] 11E4.A.1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R] 11E3.I.1. Demonstrate an understanding of compound interest. [CN, ME, PS, T] 11E3.I.2. Demonstrate an understanding of credit options, including ■ credit cards ■ loans [CN, ME, PS, R]	12E5.A.1. Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R] 12E6.A.1. Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R] 12E5.V.1. Solve problems that involve the acquisition, operation, and maintenance of a vehicle when ■ buying ■ leasing ■ leasing to buy [C, CN, PS, R, T] 12E6.B.1. Critique the viability of small business options by considering ■ expenses ■ sales ■ profit or loss [C, CN, R]

Essential Mathematics

Number

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Develop number sense.	General Learning Outcome Develop number sense.			
Specific Learning Outcomes	Specific Learning Outcomes		Specific Learning Outcomes	Specific Learning Outcomes
8.N.6. Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically. [C, CN, ME, PS] 8.N.7. Demonstrate an understanding of multiplication and division of integers, concretely, pictorially, and symbolically. [C, CN, PS, R, V] 8.N.8. Solve problems involving positive rational numbers. [C, CN, ME, PS, R, T, V]	9.N.3. Demonstrate an understanding of rational numbers by ■ comparing and ordering rational numbers ■ solving problems that involve arithmetic operations on rational numbers [C, CN, ME, PS, R, T, V] 9.N.4. Explain and apply the order of operations, including exponents, with and without technology. [ME, PS, T] 9.N.5. Determine the square root of positive rational numbers that are perfect squares. [C, CN, ME, PS, R, T] 9.N.6. Determine an approximate square root of positive rational numbers that are non-perfect squares. [C, CN, ME, PS, R, T]		11E3.I.3. Solve problems that require the manipulation and application of formulas related to ■ simple interest ■ finance charges [CN, PS, R] 11E4.M.1. Solve problems that involve personal budgets. [CN, PS, R, T] 11E4.M.2. Demonstrate an understanding of financial institution services used to access and manage finances. [C, CN, R, T] 11E4.R.2. Solve problems by applying proportional reasoning and unit analysis. [C, CN, PS, R]	12E6.B.2. Demonstrate an awareness of the government taxation forms and procedures involved in owning a business. [C, CN] 12E6.H.1. Solve problems involving the purchase and maintenance of a house. [C, CN, ME, R, T] 12E5.C.1. Create a plan for the future, including possible career choices and their requirements. [C, CN, PS, R]

Essential Mathematics

Patterns and Relations (Patterns)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Use patterns to describe the world and solve problems.	General Learning Outcome Use patterns to describe the world and solve problems.			
Specific Learning Outcomes	Specific Learning Outcomes		Specific Learning Outcomes	
8.PR.1. Graph and analyze two-variable linear equations. [C, ME, PS, R, T, V]	9.PR.1. Generalize a pattern arising from a problem-solving context using linear equations, and verify by substitution. [C, CN, PS, R, V] 9.PR.2. Graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems. [C, CN, ME, PS, R, T, V]		11E4.R.5. Demonstrate an understanding of linear relations by ■ recognizing patterns and trends ■ graphing ■ creating tables of values ■ writing equations ■ interpolating and extrapolating ■ solving problems [CN, PS, R, T, V]	

Essential Mathematics

Patterns and Relations (Variables and Equations)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Represent algebraic expressions in multiple ways.	General Learning Outcome Represent algebraic expressions in multiple ways.			
Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	
8.PR.2. Model and solve problems using linear equations of the form ■ $ax = b$ ■ $\frac{x}{a} = b, a \neq 0$ ■ $ax + b = c$ ■ $\frac{x}{a} + b = c, a \neq 0$ ■ $a(x + b) = c$ concretely, pictorially, and symbolically, where a, b, and c are integers. [C, CN, PS, V]	9.PR.3. Model and solve problems using linear equations of the form ■ $ax = b$ ■ $ax + b = c$ ■ $ax = b + cx$ ■ $a(x + b) = c$ ■ $ax + b = cx + d$ ■ $a(bx + c) = d(ex + f)$ ■ $\frac{a}{x} = b, x \neq 0$ where a, b, c, d, e, and f are rational numbers. [C, CN, ME, PS, V] 9.PR.4. Explain and illustrate strategies to solve single variable linear inequalities with rational number coefficients within a problem-solving context. [C, CN, ME, PS, R, V] 9.PR.5. Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). [C, CN, R, V]	10E1.P.2. Solve problems that require the manipulation and application of formulas related to income. [C, CN, ME, PS, R] 10E1.M.4. Solve problems that require the manipulation and application of formulas related to converting measurement. [C, CN, ME, PS, R] 10E1.G.2. Solve problems that require the manipulation and application of formulas related to ■ perimeter ■ area [C, CN, ME, PS, R]	11E3.I.3. Solve problems that require the manipulation and application of formulas related to ■ simple interest ■ finance charges [CN, PS, R] 11E3.G.3. Solve problems that require the manipulation and application of formulas related to ■ volume and capacity ■ surface area [CN, PS, R] 11E4.R.3. Solve problems that require the manipulation and application of formulas related to slope and rate of change. [CN, PS, R]	

Essential Mathematics

Patterns and Relations (Variables and Equations) *(continued)*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
	General Learning Outcome Represent algebraic expressions in multiple ways.			
	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	
	9.PR.6. Model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2). [C, CN, ME, PS, R, V] 9.PR.7. Model, record, and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially, and symbolically. [C, CN, R, V]	10E2.TG.3. Solve problems that require the manipulation and application of formulas related to ■ the Pythagorean theorem ■ primary trigonometric ratios [C, CN, ME, PS, R]	11E4.R.5. Demonstrate an understanding of linear relations by ■ recognizing patterns and trends ■ graphing ■ creating tables of values ■ writing equations ■ interpolating and extrapolating ■ solving problems [CN, PS, R, T, V]	

Essential Mathematics

Patterns and Relations (Relations and Functions)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
---------	---------	----------	----------	----------

Specific Learning Outcomes

- 11E4.R 1. Demonstrate an understanding of slope
- as rise over run
 - as rate of change
- by solving problems.
[C, CN, PS, V]
- 11E4.R 2. Solve problems by applying proportional reasoning and unit analysis.
[C, CN, PS, R]
- 11E4.R.3. Solve problems that require the manipulation and application of formulas related to slope and rate of change.
[CN, PS, R]

Essential Mathematics

Shape and Space (Measurement)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Use direct or indirect measurement to solve problems.	General Learning Outcome Use direct or indirect measurement to solve problems.			
Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes
8.SS.1. Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V] 8.SS.2. Draw and construct nets for 3-D objects. [C, CN, PS, V] 8.SS.3. Determine the surface area of ■ right rectangular prisms ■ right triangular prisms ■ right cylinders to solve problems. [C, CN, PS, R, V] 8.SS.4. Develop and apply formulas for determining the volume of right prisms and right cylinders. [C, CN, PS, R, V]	9.SS.1. Solve problems and justify the solution strategy using circle properties, including ■ the perpendicular from the centre of a circle to a chord bisects the chord ■ the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc ■ the inscribed angles subtended by the same arc are congruent ■ a tangent to a circle is perpendicular to the radius at the point of tangency [C, CN, PS, R, T, V]	10E1.M.1. Demonstrate an understanding of the Système International (SI) by describing the relationships of the units for length, area, volume, capacity, and mass. [C, CN, ME, V] 10E1.M.2. Demonstrate an understanding of the imperial system by ■ describing the relationships of the units for length, area, volume, capacity, and mass ■ comparing the American and British imperial units for capacity ■ applying strategies to convert between imperial units and SI units [C, CN, ME, V]	11E3.G.1. Solve problems that involve SI and imperial units in surface area measurements. [C, CN, ME, PS, V] 11E3.G.2. Solve problems that involve SI and imperial units in volume and capacity measurements. [C, CN, ME, PS, V] 11E3.G.3. Solve problems that require the manipulation and application of formulas related to ■ volume and capacity ■ surface area [CN, PS, R]	12E5.P.1. Demonstrate an understanding of the limitations of measuring instruments, including ■ precision ■ accuracy ■ uncertainty ■ tolerance [C, PS, R, T, V] 12E6.G.1. Solve problems by using the sine law and cosine law, excluding the ambiguous case. [CN, PS, V]

Essential Mathematics

Shape and Space (Measurement) *(continued)*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
		General Learning Outcome Use direct or indirect measurement to solve problems.	General Learning Outcome Use direct or indirect measurement to solve problems.	
		Specific Learning Outcomes	Specific Learning Outcomes	
		10E1.M.3. Solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements. [CN, ME, PS, V]	11E4.TG.1. Solve problems that involve two and three right triangles. [CN, PS, T, V]	
		10E1.M.4. Solve problems that require the manipulation and application of formulas related to converting measurement. [C, CN, ME, PS, R]	11E4.R.4. Solve problems that involve scale. [PS, R, T, V]	
		10E2.TG.1. Solve problems involving right triangles using the Pythagorean theorem. [C, CN, PS, V]		

Essential Mathematics

Shape and Space (Measurement) *(continued)*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
Specific Learning Outcomes				
		10E2.TG.2. Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by ■ applying similarity to right triangles ■ generalizing patterns from similar right triangles ■ solving problems [CN, PS, R, T, V] 10E2.TG.3. Solve problems that require the manipulation and application of formulas related to ■ the Pythagorean theorem ■ primary trigonometric ratios [C, CN, ME, PS, R] 10E2.AC.2. Solve problems that involve parallel, perpendicular, and transversal lines, and pairs of angles formed between them. [C, CN, PS, V]		

Essential Mathematics

Shape and Space (3-D Objects and 2-D Shapes)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.	General Learning Outcome Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.			
Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes
8.SS.5. Draw and interpret top, front, and side views of 3-D objects composed of right rectangular prisms. [C, CN, R, T, V]	9.SS.2. Determine the surface area of composite 3-D objects to solve problems. [C, CN, ME, PS, R, V] 9.SS.3. Demonstrate an understanding of similarity of polygons. [C, CN, PS, R, T, V]	10E1.G.1. Solve problems that involve SI and imperial area measurements of regular, composite, and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements. [ME, PS, R, V] 10E1.A.1. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R] 10E2.A.1. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	11E4.D.1. Model and draw 3-D objects and their views. [CN, R, V] 11E4.D.2. Draw and describe exploded views, component parts, and scale diagrams of simple 3-D objects. [CN, V]	12E6.G.2. Solve problems that involve ■ triangles ■ quadrilaterals ■ regular polygons [C, CN, PS, V]

Essential Mathematics

Shape and Space (3-D Objects and 2-D Shapes) *(continued)*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
---------	---------	----------	----------	----------

Specific Learning Outcomes

10E2.AC.1. Demonstrate an understanding of angles, including acute, right, obtuse, straight, and reflex, by

- drawing
- replicating and constructing
- bisecting
- solving problems

[C, ME, PS, T, V]

Essential Mathematics

Shape and Space (Transformations)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Describe and analyze position and motion of objects and shapes.	General Learning Outcome Describe and analyze position and motion of objects and shapes.			
Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes		
8.SS.6. Demonstrate an understanding of tessellation by ■ explaining the properties of shapes that make tessellating possible ■ creating tessellations ■ identifying tessellations in the environment [C, CN, PS, T, V]	9.SS.4. Draw and interpret scale diagrams of 2-D shapes. [CN, R, T, V] 9.SS.5. Demonstrate an understanding of line and rotation symmetry. [C, CN, PS, V]	10E2.TF.1. Demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including ■ translations ■ rotations ■ reflections ■ dilations [C, CN, R, T, V]		

Essential Mathematics

Statistics and Probability (Data Analysis)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Collect, display, and analyze data to solve problems.	General Learning Outcome Collect, display, and analyze data to solve problems.			
Specific Learning Outcomes	Specific Learning Outcomes		Specific Learning Outcomes	Specific Learning Outcomes
8.SP.1. Critique ways in which data are presented. [C, R, T, V]	9.SP.1. Describe the effect of ■ bias ■ use of language ■ ethics ■ cost ■ time and timing ■ privacy ■ cultural sensitivity on the collection of data. [C, CN, R, T] 9.SP.2. Select and defend the choice of using either a population or a sample of a population to answer a question. [C, CN, PS, R]		11E3.S.1. Solve problems that involve creating and interpreting graphs, including ■ bar graphs ■ histograms ■ line graphs ■ circle graphs [C, CN, PS, R, T, V]	12E5.S.1. Solve problems that involve measures of central tendency, including ■ mean ■ median ■ mode ■ weighted mean ■ trimmed mean [C, CN, PS, R] 12E5.S.2. Analyze and describe percentiles. [C, CN, PS, R]

Essential Mathematics

Statistics and Probability (Data Analysis) *(continued)*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Collect, display, and analyze data to solve problems.				
Specific Learning Outcomes				
9.SP. 3. Develop and implement a project plan for the collection, display, and analysis of data by ■ formulating a question for investigation ■ choosing a data collection method that includes social considerations ■ selecting a population or a sample ■ collecting the data ■ displaying the collected data in an appropriate manner ■ drawing conclusions to answer the question [C, PS, R, T, V]				

Essential Mathematics

Statistics and Probability (Chance and Uncertainty)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.	General Learning Outcome Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.			
Specific Learning Outcomes	Specific Learning Outcomes			Specific Learning Outcomes
8.SP.2. Solve problems involving the probability of independent events. [C, CN, PS, T]	9.SP.4. Demonstrate an understanding of the role of probability in society. [C, CN, R, T]			12E6.P.1. Analyze and interpret problems that involve probability. [C, CN, PS, R]

General and Specific Learning Outcomes by Strand

Pre-Calculus Mathematics

Number

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Develop number sense.	General Learning Outcome Develop number sense.			
Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes
8.N.1. Demonstrate an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers). [C, CN, R, V] 8.N.2. Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). [C, CN, ME, R, T] 8.N.3. Demonstrate an understanding of percents greater than or equal to 0%. [CN, PS, R, V] 8.N.4. Demonstrate an understanding of ratio and rate. [C, CN, V] 8.N.5. Solve problems that involve rates, ratios, and proportional reasoning. [C, CN, PS, R]	9.N.1. Demonstrate an understanding of powers with integral bases (excluding base 0) and whole-number exponents by ■ representing repeated multiplication using powers ■ using patterns to show that a power with an exponent of zero is equal to one ■ solving problems involving powers [C, CN, ME, PS, R] 9.N.2. Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole-number exponents. [C, CN, ME, PS, R, T]	10I.A.1. Demonstrate an understanding of factors of whole numbers by determining ■ prime factors ■ greatest common factor ■ least common multiple ■ square root ■ cube root [CN, ME, R] 10I.A.2. Demonstrate an understanding of irrational numbers by ■ representing, identifying, and simplifying irrational numbers ■ ordering irrational numbers [CN, ME, R, V] 10I.A.3. Demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R]	11P.A.1. Demonstrate an understanding of the absolute value of real numbers. [ME, R, V] 11P.A.2. Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands. [CN, ME, PS, R, T]	12P.P.1. Apply the fundamental counting principle to solve problems. [C, CN, PS, R, V] 12P.P.2. Determine the number of permutations of n elements taken r at a time to solve problems. [C, PS, R, V] 12P.P.3. Determine the number of combinations of n different elements taken r at a time to solve problems. [C, PS, R, V]

Pre-Calculus Mathematics

Number (*continued*)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Develop number sense.	General Learning Outcome Develop number sense.			
Specific Learning Outcomes	Specific Learning Outcomes			
8.N.6. Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically. [C, CN, ME, PS]	9.N.3. Demonstrate an understanding of rational numbers by ■ comparing and ordering rational numbers ■ solving problems that involve arithmetic operations on rational numbers [C, CN, ME, PS, R, T, V]			
8.N.7. Demonstrate an understanding of multiplication and division of integers, concretely, pictorially, and symbolically. [C, CN, PS, R, V]	9.N.4. Explain and apply the order of operations, including exponents, with and without technology. [ME, PS, T]			
8.N.8. Solve problems involving positive rational numbers. [C, CN, ME, PS, R, T, V]	9.N.5. Determine the square root of positive rational numbers that are perfect squares. [C, CN, ME, PS, R, T]			
	9.N.6. Determine an approximate square root of positive rational numbers that are non-perfect squares. [C, CN, ME, PS, R, T]			

Pre-Calculus Mathematics

Patterns and Relations (Patterns)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Use patterns to describe the world and solve problems.	General Learning Outcome Use patterns to describe the world and solve problems.			
Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes
8.PR.1. Graph and analyze two-variable linear equations. [C, ME, PS, R, T, V]	9.PR.1. Generalize a pattern arising from a problem-solving context using linear equations, and verify by substitution. [C, CN, PS, R, V] 9.PR.2. Graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems. [C, CN, ME, PS, R, T, V]	10.I.R.1. Interpret and explain the relationships among data, graphs, and contexts. [C, CN, R, T, V]	11.P.R.9. Analyze arithmetic sequences and series to solve problems. [C, CN, PS, R, T] 11.P.R.10. Analyze geometric sequences and series to solve problems. [C, CN, PS, R, T]	12.P.T.6. Prove trigonometric identities, using ■ reciprocal identities ■ quotient identities ■ Pythagorean identities ■ sum or difference identities (restricted to sine, cosine, and tangent) ■ double-angle identities (restricted to sine, cosine, and tangent) [C, R, T, V]

Pre-Calculus Mathematics

Patterns and Relations (Variables and Equations)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Represent algebraic expressions in multiple ways.	General Learning Outcome Represent algebraic expressions in multiple ways.			
Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes
8.PR.2. Model and solve problems using linear equations of the form ■ $ax = b$ ■ $\frac{x}{a} = b, a \neq 0$ ■ $ax + b = c$ ■ $\frac{x}{a} + b = c, a \neq 0$ ■ $a(x + b) = c$ concretely, pictorially, and symbolically, where a, b, and c are integers. [C, CN, PS, V]	9.PR.3. Model and solve problems using linear equations of the form ■ $ax = b$ ■ $ax + b = c$ ■ $ax = b + cx$ ■ $a(x + b) = c$ ■ $ax + b = cx + d$ ■ $a(bx + c) = d(ex + f)$ ■ $\frac{a}{x} = b, x \neq 0$ where a, b, c, d, e, and f are rational numbers. [C, CN, ME, PS, V] 9.PR.4. Explain and illustrate strategies to solve single variable linear inequalities with rational number coefficients within a problem-solving context. [C, CN, ME, PS, R, V]	10I.A.3. Demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R] 10I.A.4. Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials), concretely, pictorially, and symbolically. [C, CN, R, V] 10I.A.5. Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially, and symbolically. [C, CN, R, V]	11P.A.2. Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands. [CN, ME, PS, R, T] 11P.A.3. Solve problems that involve radical equations (limited to square roots). [C, CN, PS, R, T] 11P.A.4. Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials). [C, ME, R] 11P.A.5. Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials). [C, CN, ME, R]	12P.P.4. Expand powers of a binomial in a variety of ways, including using the binomial theorem (restricted to exponents that are natural numbers). [C, CN, R, V] 12P.T.5. Solve, algebraically and graphically, first and second degree trigonometric equations with the domain expressed in degrees and radians. [C, CN, PS, R, T, V]

Pre-Calculus Mathematics

Patterns and Relations (Variables and Equations) *(continued)*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
	General Learning Outcome Represent algebraic expressions in multiple ways.			
	Specific Learning Outcomes		Specific Learning Outcomes	
	9.PR.5. Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). [C, CN, R, V] 9.PR.6. Model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2). [C, CN, ME, PS, R, V] 9.PR.7. Model, record, and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially, and symbolically. [C, CN, R, V]		11P.A.6. Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials, or trinomials). [C, CN, PS, R] 11P.R.1. Factor polynomial expressions of the form ■ $ax^2 + bx + c, a \neq 0$ ■ $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$ ■ $a(f(x))^2 + b(f(x)) + c, a \neq 0$ ■ $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$ where a, b, and c are rational numbers. [ME, R]	

Pre-Calculus Mathematics

Patterns and Relations (Relations and Functions)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
		Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes
		10I.R.1. Interpret and explain the relationships among data, graphs, and contexts. [C, CN, R, T, V] 10I.R.2. Demonstrate an understanding of relations and functions. [C, R, V] 10I.R.3. Demonstrate an understanding of slope with respect to ■ rise and run ■ line segments and lines ■ rate of change ■ parallel lines ■ perpendicular lines [PS, R, V]	11P.R.2. Graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems. [C, PS, R, T, V] 11P.R.3. Analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the ■ vertex ■ domain and range ■ direction of opening ■ axis of symmetry ■ x- and y-intercepts [C, CN, R, T, V]	12P.T.2. Develop and apply the equation of the unit circle. [CN, R, V] 12P.T.3. Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees. [C, ME, PS, R, T, V] 12P.T.4. Graph and analyze the trigonometric functions sine, cosine, and tangent to solve problems. [C, CN, PS, T, V]

Pre-Calculus Mathematics

Patterns and Relations (Relations and Functions) *(continued)*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
		Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes
		10I.R.4. Describe and represent linear relations, using ■ words ■ ordered pairs ■ tables of values ■ graphs ■ equations [C, CN, R, V] 10I.R.5. Determine the characteristics of the graphs of linear relations, including the ■ intercepts ■ slope ■ domain ■ range [CN, PS, R, T, V]	11P.R.4. Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including ■ vertex ■ domain and range ■ direction of opening ■ axis of symmetry ■ x- and y-intercepts [C, CN, PS, R, T, V] 11P.R.5. Solve problems that involve quadratic equations. [C, CN, PS, R, T, V] 11P.R.6. Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables. [C, CN, PS, R, T, V]	12P.R.1. Demonstrate an understanding of operations on, and compositions of, functions. [CN, R, T, V] 12P.R.2. Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations. [C, CN, R, V] 12P.R.3. Demonstrate an understanding of the effects of horizontal and vertical compressions and stretches on the graphs of functions and their related equations. [C, CN, R, V] 12P.R.4. Apply translations, compressions, and stretches to the graphs and equations of functions. [C, CN, R, V]

Pre-Calculus Mathematics

Patterns and Relations (Relations and Functions) *(continued)*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
		Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes
		10I.R.6. Relate linear relations expressed in ■ slope–intercept form ($y = mx + b$) ■ general form ($Ax + By + C = 0$) ■ slope–point form ($y - y_1 = m(x - x_1)$) to their graphs. [C, CN, R, T, V] 10I.R.7. Determine the equation of a linear relation, given ■ a graph ■ a point and the slope ■ two points ■ a point and the equation of a parallel or perpendicular line ■ a scatterplot [C, CN, PS, R, T, V]	11P.R.7. Solve problems that involve linear and quadratic inequalities in two variables. [C, PS, T, V] 11P.R.8. Solve problems that involve quadratic inequalities in one variable. [CN, PS, V] 11P.R.11. Graph and analyze reciprocal functions (limited to the reciprocal of linear and quadratic functions). [CN, R, T, V]	12P.R.5. Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections through the ■ x-axis ■ y-axis ■ line $y = x$ [C, CN, R, V] 12P.R.6. Demonstrate an understanding of inverses of relations. [C, CN, R, V] 12P.R.7. Demonstrate an understanding of logarithms. [C, CN, ME, R]

Pre-Calculus Mathematics

Patterns and Relations (Relations and Functions) *(continued)*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
		Specific Learning Outcomes		Specific Learning Outcomes
		10I.R.8. Represent a linear function, using function notation. [CN, ME, V]		12P.R.8. Demonstrate an understanding of the product, quotient, and power laws of logarithms. [C, CN, R, T]
		10I.R.9. Solve problems that involve systems of linear equations in two variables, graphically and algebraically. [CN, PS, R, T, V]		12P.R.9. Graph and analyze exponential and logarithmic functions. [C, CN, T, V]
		10I.R.10. Solve problems that involve the distance between two points and the midpoint of a line segment. [C, CN, PS, V]		12P.R.10. Solve problems that involve exponential and logarithmic equations. [C, CN, PS, R]
				12P.R.11. Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree ≤ 5 with integral coefficients). [C, CN, ME]

Pre-Calculus Mathematics

Patterns and Relations (Relations and Functions) (*continued*)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
				Specific Learning Outcomes
				12P.R.12. Graph and analyze polynomial functions (limited to polynomial functions of degree ≤ 5). [C, CN, PS, T, V]
				12P.R.13. Graph and analyze radical functions (limited to functions involving one radical). [C, CN, R, T, V]
				12P.R.14. Graph and analyze rational functions (limited to numerators and denominators that are monomials, binomials, or trinomials). [C, CN, R, T, V]

Pre-Calculus Mathematics

Shape and Space (Measurement)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Use direct or indirect measurement to solve problems.	General Learning Outcome Use direct or indirect measurement to solve problems.			
Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes
8.SS.1. Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V] 8.SS.2. Draw and construct nets for 3-D objects. [C, CN, PS, V] 8.SS.3. Determine the surface area of ■ right rectangular prisms ■ right triangular prisms ■ right cylinders to solve problems. [C, CN, PS, R, V] 8.SS.4. Develop and apply formulas for determining the volume of right prisms and right cylinders. [C, CN, PS, R, V]	9.SS.1. Solve problems and justify the solution strategy using circle properties, including ■ the perpendicular from the centre of a circle to a chord bisects the chord ■ the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc ■ the inscribed angles subtended by the same arc are congruent ■ a tangent to a circle is perpendicular to the radius at the point of tangency [C, CN, PS, R, T, V]	10I.M.1. Solve problems that involve linear measurement, using ■ SI and imperial units of measure ■ estimation strategies ■ measurement strategies [ME, PS, V] 10I.M.2. Apply proportional reasoning to problems that involve conversions between SI and imperial units of measure. [C, ME, PS, T]	11P.T.1. Demonstrate an understanding of angles in standard position [0° to 360°]. [C, R, V] 11P.T.2. Solve problems, using the three primary trigonometric ratios (sine, cosine, and tangent) for angles from 0° to 360° in standard position. [C, ME, PS, R, T, V] 11P.T.3. Solve problems, using the cosine law and sine law, including the ambiguous case. [C, CN, PS, R, T]	12P.T.1. Demonstrate an understanding of angles in standard position, expressed in degrees and radians. [CN, ME, R, V]

Pre-Calculus Mathematics

Shape and Space (3-D Objects and 2-D Shapes)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.	General Learning Outcome Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.			
Specific Learning Outcomes	Specific Learning Outcomes	Specific Learning Outcomes		
8.SS.5. Draw and interpret top, front, and side views of 3-D objects composed of right rectangular prisms. [C, CN, R, T, V]	9.SS.2. Determine the surface area of composite 3-D objects to solve problems. [C, CN, ME, PS, R, V] 9.SS.3. Demonstrate an understanding of similarity of polygons. [C, CN, PS, R, T, V]	10I.M.3. Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including ■ right cones ■ right cylinders ■ right prisms ■ right pyramids ■ spheres [CN, PS, R, T, V] 10I.M.4. Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles. [C, CN, PS, R, T, V]		

Pre-Calculus Mathematics

Shape and Space (Transformations)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Describe and analyze position and motion of objects and shapes.	General Learning Outcome Describe and analyze position and motion of objects and shapes.			
Specific Learning Outcomes	Specific Learning Outcomes			
8.SS.6. Demonstrate an understanding of tessellation by ■ explaining the properties of shapes that make tessellating possible ■ creating tessellations ■ identifying tessellations in the environment [C, CN, PS, T, V]	9.SS.4. Draw and interpret scale diagrams of 2-D shapes. [CN, R, T, V] 9.SS.5. Demonstrate an understanding of line and rotation symmetry. [C, CN, PS, V]			

Pre-Calculus Mathematics

Statistics and Probability (Data Analysis)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Collect, display, and analyze data to solve problems.	General Learning Outcome Collect, display, and analyze data to solve problems.			
Specific Learning Outcomes	Specific Learning Outcomes			
8.SP.1. Critique ways in which data are presented. [C, R, T, V]	9.SP.1. Describe the effect of ■ bias ■ use of language ■ ethics ■ cost ■ time and timing ■ privacy ■ cultural sensitivity on the collection of data. [C, CN, R, T] 9.SP.2. Select and defend the choice of using either a population or a sample of a population to answer a question. [C, CN, PS, R]			

Pre-Calculus Mathematics

Statistics and Probability (Data Analysis) *(continued)*

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Collect, display, and analyze data to solve problems.				
Specific Learning Outcomes				
9.SP. 3. Develop and implement a project plan for the collection, display, and analysis of data by ■ formulating a question for investigation ■ choosing a data collection method that includes social considerations ■ selecting a population or a sample ■ collecting the data ■ displaying the collected data in an appropriate manner ■ drawing conclusions to answer the question [C, PS, R, T, V]				

Pre-Calculus Mathematics

Statistics and Probability (Chance and Uncertainty)

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
General Learning Outcome Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.	General Learning Outcome Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.			
Specific Learning Outcomes	Specific Learning Outcomes			
8.SP.2. Solve problems involving the probability of independent events. [C, CN, PS, T]	9.SP.4. Demonstrate an understanding of the role of probability in society. [C, CN, R, T]			

GENERAL AND SPECIFIC LEARNING OUTCOMES WITH ACHIEVEMENT INDICATORS BY COURSE

This section presents specific learning outcomes with corresponding achievement indicators for each of the nine courses in Grades 9 to 12 Mathematics. Achievement indicators describe the depth and scope of each specific learning outcome. The list of indicators contained in this document is not intended to be exhaustive but rather to provide teachers with examples of evidence of understanding that may be used in determining whether or not students understand a given learning outcome. They are not presented in any particular order and need not be explicitly addressed in the classroom. Teachers may use any number of these indicators, or they may choose to use other indicators as evidence that the desired learning has been achieved. However, students need to understand the learning outcomes at least to the depth indicated by the indicators. Therefore the achievement indicators are sufficient as a basis for instructional design and assessment, and will form the basis for provincial assessment as appropriate.

Grade 9 Mathematics (10F)

Grade 9 Mathematics (10F) is a foundation course to prepare students for multiple possible pathways in Grades 10 to 12. The course builds on the understandings from Kindergarten to Grade 8 Mathematics (for details, please see *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes*).

The activities that take place in the Grade 9 Mathematics classroom should stem from a problem-solving approach and be based on the seven mathematical processes. Students should develop an understanding of the nature of mathematics through specific knowledge, skills, and connections among and between strands.

The general focus in most units should be to allow time for hands-on activities that promote concrete understanding of concepts.

A focus on developing problem-solving skills will enable students to move on with a deeper understanding of mathematics. The emphasis should be on “why” and not just “how.”

The learning outcomes are divided into four strands: Number; Patterns and Relations; Shape and Space; Statistics and Probability. For instructional purposes, the outcomes could be arranged into units. Learning outcomes from different strands could be taught in the same unit. Some learning outcomes may fit into multiple units and parts of the learning outcome could be taught in one unit while the remaining parts can be taught later. Two possible sequences of the learning outcomes into units with suggested time allotments follow. These are not the only possibilities but will provide some direction for those who are teaching this course for the first time. The time for each unit includes instructional and assessment time.

Regardless of the organization of the learning outcomes into units, students should constantly be looking for and be given opportunities to see connections between the various learning outcomes in Grade 9 Mathematics.

Assessment of Grade 9 Mathematics should be a balance of assessment *for* learning, assessment *as* learning, and assessment *of* learning. Assessment tools used in Grade 9 Mathematics should be varied and may include

observation, homework, learning conversations or interviews, summative unit essays, demonstrations, presentations, performance tasks, learning logs, projects, investigations, reflective journals, portfolios, quizzes, tests, and examinations. An appropriately prepared portfolio requires a consistent effort throughout the school term and a commitment to completing quality work on a daily basis.

Possibility 1			Possibility 2		
Unit	Learning Outcomes	Suggested Hours	Unit	Learning Outcomes	Suggested Hours
Number Sense	N3, PR3, PR4, SP4, N5, N6	25	Circle Geometry	SS1	10
Statistics	SP1, SP2, SP3	15	Number Sense	N3, N5, N6	15
Powers	N1, N2, N4	15	Polynomials	PR5, PR6, PR7	15
Linear Relations	PR1, PR2, SP4	10	Powers	N1, N2, N4	10
Circle Geometry	SS1	10	Linear Relations	PR1, PR2, PR3, PR4	20
Polynomials	PR5, PR6, PR7	15	Statistics and Probability	SP1, SP2, SP3, SP4	20
Similarity	SS2, SS3, SS4	15	Symmetry	SS5	5
Symmetry	SS5	5	Geometry	SS2, SS3, SS4	15
Total		110	Total		110

General and Specific Learning Outcomes with Achievement Indicators by Course

Grade 9

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Number		General Learning Outcome: Develop number sense.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
9.N.1.	Demonstrate an understanding of powers with integral bases (excluding base 0) and whole-number exponents by ■ representing repeated multiplication using powers ■ using patterns to show that a power with an exponent of zero is equal to one ■ solving problems involving powers [C, CN, ME, PS, R]	■ Demonstrate the differences between the exponent and the base by building models of a power, such as 2^3 and 3^2. ■ Explain, using repeated multiplication, the difference between two powers in which the exponent and base are interchanged, such as 10^3 and 3^{10}. ■ Express a power as a repeated multiplication. ■ Express a repeated multiplication as a power. ■ Explain the role of parentheses in powers by evaluating a set of powers such as $(-2)^4$, (-2^4), and -2^4. ■ Demonstrate in a variety of ways that a^0 is equal to 1 for any value of a ($a \neq 0$). ■ Evaluate powers with integral bases (excluding base 0) and whole-number exponents. ■ Determine the sum of two powers such as $5^2 + 5^3$ or $3^2 + 2^3$, and record the process. ■ Determine the difference of two powers such as $4^3 - 4^2$, and record the process.
9.N.2.	Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole-number exponents. [C, CN, ME, PS, R, T] NOTE a & b are non-zero integers and m & n are whole numbers.	■ Explain, using examples, the exponent laws of powers with integral bases (excluding base 0) and whole-number exponents: ■ $(a^m)(a^n) = a^{m+n}$ ■ $a^m \div a^n = a^{m-n}, m \geq n$ ■ $(a^m)^n = a^{mn}$ ■ $(ab)^m = a^m b^m$ ■ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$ ■ Evaluate an expression by applying the exponent laws. ■ Evaluate an expression where the exponent laws do not apply. ■ Identify the error(s) in a simplification of an expression involving powers.

Grade 9

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Number (continued)		General Learning Outcome: Develop number sense.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
9.N.3.	Demonstrate an understanding of rational numbers by ■ comparing and ordering rational numbers ■ solving problems that involve arithmetic operations on rational numbers [C, CN, ME, PS, R, T, V]	■ Order a set of rational numbers, in fraction or decimal form, by placing them on a vertical or horizontal number line (e.g., $\frac{3}{5}$, $-0.666\dots$, 0.5, $-\frac{5}{8}$). ■ Identify a rational number that is between two rational numbers. ■ Solve a problem involving operations on rational numbers in fraction form, decimal form, or a combination of rational forms.
9.N.4.	Explain and apply the order of operations, including exponents, with and without technology. [ME, PS, T]	■ Demonstrate and explain with examples the need for a standardized order of operations. ■ Solve a problem by applying the order of operations without the use of technology, and record the process. ■ Solve a problem by applying the order of operations with the use of technology, and record the process. ■ Identify the error in applying the order of operations in an incorrect solution.
9.N.5.	Determine the square root of positive rational numbers that are perfect squares. [C, CN, ME, PS, R, T]	■ Determine whether or not a rational number is a square number, and explain the reasoning. ■ Determine the square root of a positive rational number that is a perfect square. ■ Identify the error made in a calculation of a square root. ■ Determine a positive rational number given the square root of that positive rational number. ■ Explain with examples why a positive rational number has both a positive and a negative square root.
9.N.6.	Determine an approximate square root of positive rational numbers that are non-perfect squares. [C, CN, ME, PS, R, T]	■ Estimate the square root of a rational number that is not a perfect square, using the roots of perfect squares as benchmarks. ■ Determine an approximate square root of a rational number that is not a perfect square, using technology. ■ Explain why the square root of a rational number as shown on a calculator may be an approximation. ■ Identify a number with a square root that is between two numbers.

Grade 9

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Patterns and Relations (Patterns)		General Learning Outcome: Use patterns to describe the world and solve problems.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
9.PR.1.	Generalize a pattern arising from a problem-solving context using linear equations, and verify by substitution. [C, CN, PS, R, V]	■ Write an expression representing a pictorial, oral, or written pattern. ■ Write a linear equation to represent a context. ■ Create a context for a linear equation. ■ Solve, using a linear equation, a problem that involves pictorial, oral, or written linear patterns. ■ Write a linear equation representing the pattern in a table of values, and verify the equation by substituting values from the table.
9.PR.2.	Graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems. [C, CN, ME, PS, R, T, V]	■ Describe a pattern found in a graph. ■ Graph a linear relation from a table of values or from a context ■ Match a contextual situation to a linear relation or graph. ■ Extend a graph (extrapolate) to determine the value of an unknown element. ■ Interpolate the approximate value of one variable on a graph given the value of the other variable. ■ Extrapolate the approximate value of one variable from a graph given the value of the other variable. ■ Solve a problem by graphing a linear relation and analyzing the graph.

Grade 9

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Patterns and Relations (Variables and Equations)		General Learning Outcome: Represent algebraic expressions in multiple ways.
Specific Learning Outcomes <i>It is expected that students will:</i>	Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>	
9.PR.3. Model and solve problems using linear equations of the form ■ $ax = b$ ■ $ax + b = c$ ■ $ax = b + cx$ ■ $a(x + b) = c$ ■ $ax + b = cx + d$ ■ $a(bx + c) = d(ex + f)$ ■ $\frac{a}{x} = b, x \neq 0$ where a, b, c, d, e , and f are rational numbers. [C, CN, ME, PS, V] It is intended that this learning outcome build on the prior work of solving linear equations with integral coefficients.	■ Model the solution of a linear equation using concrete or pictorial representations, and record the process. ■ Model the use of preservation of equality when solving linear equations, concretely, pictorially, or symbolically. ■ Determine, by substitution, whether a rational number is a solution to a linear equation. ■ Develop a list of strategies for solving any linear equation. ■ Solve a linear equation symbolically. ■ Identify and correct an error in an incorrect solution of a linear equation. ■ Represent a problem using a linear equation. ■ Create a context for a linear equation. ■ Solve a problem using a linear equation, and record the process.	

Grade 9

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Patterns and Relations
(Variables and Equations) *(continued)*

General Learning Outcome:
Represent algebraic expressions in multiple ways.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 9.PR.4. Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.
[C, CN, ME, PS, R, V]

- Translate a problem into a single variable linear inequality using the symbols $\geq$, $>$, $<$, or $\leq$.
- Create a context for a linear inequality expressed graphically or symbolically.
- Determine if a rational number is a possible solution of a linear inequality.
- Generalize and apply a rule for adding or subtracting a positive or negative number to determine the solution of an inequality.
- Generalize and apply a rule for multiplying or dividing by a positive or negative number to determine the solution of an inequality.
- Solve a linear inequality algebraically, and explain the process orally or in written form.
- Graph the solution of a linear inequality on a vertical or horizontal number line.
- Compare and explain the process for solving a linear equation to the process for solving a linear inequality.
- Compare and explain the solution of a linear equation to the solution of a linear inequality.
- Verify the solution of a linear inequality using substitution for multiple elements in the solution.
- Solve a problem involving a single variable linear inequality, and graph the solution.

Grade 9

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Patterns and Relations (Variables and Equations) <i>(continued)</i>		General Learning Outcome: Represent algebraic expressions in multiple ways.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
9.PR.5.	Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). [C, CN, R, V]	■ Create a concrete model or a pictorial representation for a polynomial expression. ■ Write the expression for a model of a polynomial. ■ Identify the variables, exponent, number of terms, and coefficients, including the constant term, of a simplified polynomial expression. ■ Create a situation for a first-degree polynomial expression. ■ Match equivalent polynomial expressions in simplified form.
9.PR.6.	Model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2). [C, CN, ME, PS, R, V]	■ Model addition of two polynomial expressions concretely or pictorially, and record the process symbolically. ■ Model subtraction of two polynomial expressions concretely or pictorially, and record the process symbolically. ■ Apply a personal strategy for addition and subtraction of polynomial expressions, and explain the process. ■ Identify equivalent polynomial expressions from a set of polynomial expressions, including pictorial and symbolic representations. ■ Identify the error(s) in a simplification of a polynomial expression.
9.PR.7.	Model, record, and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially, and symbolically. [C, CN, R, V]	■ Model multiplication of a polynomial expression by a monomial, concretely or pictorially, and record the process symbolically. ■ Model division of a polynomial expression by a monomial, concretely or pictorially, and record the process symbolically. ■ Apply a personal strategy for multiplication and division of a polynomial expression by a monomial, and explain the process. ■ Provide examples of equivalent polynomial expressions. ■ Identify the error(s) in a simplification of a polynomial expression.

Grade 9

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Shape and Space (Measurement)		General Learning Outcome: Use direct or indirect measurement to solve problems.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
9.SS.1.	Solve problems and justify the solution strategy using circle properties, including ■ the perpendicular from the centre of a circle to a chord bisects the chord ■ the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc ■ the inscribed angles subtended by the same arc are congruent ■ a tangent to a circle is perpendicular to the radius at the point of tangency [C, CN, PS, R, T, V]	■ Provide an example that illustrates ■ the perpendicular from the centre of a circle to a chord bisects the chord ■ the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc ■ the inscribed angles subtended by the same arc are congruent ■ a tangent to a circle is perpendicular to the radius at the point of tangency ■ Explore the reverse relationship of the above circle properties. ■ Solve a problem involving application of one or more of the circle properties. ■ Determine the measure of any inscribed angle subtended by the diameter using the circle properties. ■ Solve a problem involving application of one or more of the circle properties. ■ Explain the relationship among the centre of a circle, a chord, and the perpendicular bisector of the chord.

Grade 9

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Shape and Space (3-D Objects and 2-D Shapes)		General Learning Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
9.SS.2.	Determine the surface area of composite 3-D objects to solve problems. [C, CN, ME, PS, R, V] It is intended that the composite 3-D objects be made up of objects limited to right cylinders, right rectangular prisms, and right triangular prisms.	■ Determine the surface area of a concrete composite 3-D object. ■ Determine the area of overlap in a concrete composite 3-D object, and explain its effect on determining the surface area. ■ Solve a problem involving surface area.
9.SS.3.	Demonstrate an understanding of similarity of polygons. [C, CN, PS, R, V]	■ Determine if the polygons in a pre-sorted set are similar, and explain the reasoning. ■ Draw a polygon similar to a given polygon, and explain why the two are similar. ■ Solve a problem using the properties of similar polygons.

Grade 9

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Shape and Space (Transformations)		General Learning Outcome: Describe and analyze position and motion of objects and shapes.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
9.SS.4.	Draw and interpret scale diagrams of 2-D shapes. [CN, R, T, V]	■ Identify an example in print and electronic media (e.g., newspapers, the Internet) of a scale diagram, and interpret the scale factor. ■ Draw a diagram to scale that represents an enlargement or reduction of a 2-D shape. ■ Determine the scale factor for a diagram drawn to scale. ■ Determine if a diagram is proportional to the original 2-D shape and, if it is, state the scale factor. ■ Solve a problem that involves a scale diagram by applying the properties of similar polygons.

Grade 9

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Shape and Space (Transformations) (<i>continued</i>)		General Learning Outcome: Describe and analyze position and motion of objects and shapes.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
9.SS.5.	Demonstrate an understanding of line and rotation symmetry. [C, CN, PS, V]	■ Classify a set of 2-D shapes or designs according to the number of lines of symmetry. ■ Complete a 2-D shape or design given one-half of the shape or design and a line of symmetry. ■ Determine if a 2-D shape or design has rotation symmetry about the point at the centre of the shape or design and, if it does, state the order and angle of rotation. ■ Rotate a 2-D shape about a vertex and draw the resulting image. ■ Identify a line of symmetry or the order and angle of rotation symmetry in a tessellation. ■ Identify and describe the types of symmetry created in a piece of artwork. ■ Create or provide a piece of artwork that demonstrates line or rotation symmetry, and identify the line(s) of symmetry and the order and angle of rotation.

Grade 9

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Statistics and Probability (Data Analysis)		General Learning Outcome: Collect, display, and analyze data to solve problems.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
9.SP.1.	Describe the effect of ■ bias ■ use of language ■ ethics ■ cost ■ time and timing ■ privacy ■ cultural sensitivity on the collection of data. [C, CN, R, T]	■ Analyze a case study of data collection, and identify potential problems related to bias, use of language, ethics, cost, privacy, or cultural sensitivity, time and timing. ■ Provide examples to illustrate how bias, use of language, ethics, cost, privacy, or cultural sensitivity, time and timing may influence the data.
9.SP.2.	Select and defend the choice of using either a population or a sample of a population to answer a question. [C, CN, PS, R]	■ Identify whether a situation represents the use of a sample or a population. ■ Provide an example of a situation in which a population may be used to answer a question, and justify the choice. ■ Provide an example of a question where a limitation precludes the use of a population, and describe the limitation (e.g., too costly, not enough time, limited resources). ■ Identify and critique an example in which a generalization from a sample of a population may or may not be valid for the population.

Grade 9

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Statistics and Probability (Data Analysis) <i>(continued)</i>		General Learning Outcome: Collect, display, and analyze data to solve problems.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
9.SP.3.	Develop and implement a project plan for the collection, display, and analysis of data by ■ formulating a question for investigation ■ choosing a data collection method that includes social considerations ■ selecting a population or a sample ■ collecting the data ■ displaying the collected data in an appropriate manner ■ drawing conclusions to answer the question [C, PS, R, T, V]	■ Create a rubric to assess a project that includes the assessment of ■ a question for investigation ■ the choice of a data collection method that includes social considerations ■ the selection of a population or a sample and justifying the choice ■ the display of the collected data ■ the conclusions to answer the question ■ Develop a project plan that describes ■ a question for investigation ■ the method of data collection that includes social considerations ■ the method for selecting a population or a sample ■ the method to be used for collection of the data ■ the methods for analysis and display of the data ■ Complete the project according to the plan, draw conclusions, and communicate findings to an audience. ■ Self-assess the completed project by applying the rubric.

Grade 9

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Statistics and Probability (Chance and Uncertainty)

General Learning Outcome:
Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

9.SP.4. Demonstrate an understanding of the role of probability in society.
[C, CN, R, T]

- Provide an example from print and electronic media (e.g., newspapers, the Internet) where probability is used.
- Identify the assumptions associated with a given probability, and explain the limitations of each assumption.
- Explain how a single probability can be used to support opposing positions.
- Explain, using examples, how decisions based on probability may be a combination of theoretical probability, experimental probability, and subjective judgment.

GRADE 10 ESSENTIAL MATHEMATICS (20S)

Grade 10 Essential Mathematics (20S) is intended for students whose post-secondary planning does not include a focus on mathematics and science-related fields. Grade 10 Essential Mathematics (20S) is a one-credit course consisting of two half-credits each emphasizing consumer applications, problem solving, decision making, and spatial sense.

Students are expected to work both individually and in small groups on mathematical concepts and skills encountered in everyday life in a technological society.

Assessment of Grade 10 Essential Mathematics should be a balance of assessment *for* learning, assessment *as* learning, and assessment *of* learning. Assessment tools used in

Grade 10 Essential Mathematics should be varied and may include observation, homework, learning conversations or interviews, summative unit essays, demonstrations, presentations, performance tasks, learning logs, projects, investigations, reflective journals, portfolios, quizzes, tests, and examinations. An appropriately prepared portfolio requires a consistent effort throughout the school term and a commitment to completing quality work on a daily basis.

The following table lists the units of study for each half-course, along with an estimated number of hours for each unit. The time for each unit includes instructional and assessment time.

Grade 10 Essential Mathematics (25S) Half Course I		Grade 10 Essential Mathematics (25S) Half Course II	
Unit	Suggested Hours	Unit	Suggested Hours
Analysis of Games and Numbers	6	Analysis of Games and Numbers	5
Personal Finance	22	Trigonometry	17
Measurement	17	Consumer Decisions	6
2-D Geometry	10	Transformations	12
		Angle Construction	15
Total	55	Total	55

General and Specific Learning Outcomes with Achievement Indicators by Course

Grade 10 Essential Mathematics – Half Course I

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Analysis of Games and Numbers		General Learning Outcome: Develop critical thinking skills.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
10E1.A.1.	Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	■ Determine, explain, and verify a strategy to solve a puzzle or to win a game such as ■ guess and check ■ look for a pattern ■ make a systematic list ■ draw or model ■ eliminate possibilities ■ simplify the original problem ■ work backward ■ develop alternative approaches ■ Identify and correct errors in a solution to a puzzle or in a strategy for winning a game. ■ Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

It is intended that this learning outcome be integrated throughout the course by using sliding, rotation, construction, deconstruction, and similar puzzles and games.

Grade 10 Essential Mathematics – Half Course I

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Personal Finance		General Learning Outcome: Develop an understanding of employment earnings.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
10E1.P.1.	Demonstrate an understanding of calculations for gross pay and net pay earned through income sources, including ■ wages ■ salary ■ contracts ■ commissions ■ piecework [C, CN, R, T]	■ Describe, using examples, various methods of earning income. ■ Identify and list jobs that commonly use different methods of earning income, such as hourly wage, wage and tips, salary, piecework, commission, contract, bonus, and shift premiums. ■ Describe the advantages and disadvantages for a method of earning income, such as hourly wage, piecework, salary, commission, and contract. ■ Determine in decimal form, from a time schedule, the total time worked in hours and minutes, separated into regular time, time and a half, or double time. ■ Describe the different ways that employers penalize workers for showing up late or missing shifts. ■ Determine gross pay from various income methods, including ■ the base hourly wage, with and without tips ■ the base hourly wage, plus daily or weekly overtime ■ base wage, plus single commission rate ■ single commission rate ■ piecework ■ contract work ■ Describe the different ways that employers deal with overtime, including non-monetary compensation. ■ Explain why gross pay and net pay are not the same. ■ Determine the Canadian Pension Plan (CPP), Employment Insurance (EI), and income tax deductions for a given gross pay. ■ Determine net pay when given CPP, EI, and income tax deductions, along with deductions such as health plans, uniforms, union dues, charitable donations, or savings plans. ■ Investigate, with technology, “what if ...” questions related to changes in income. ■ Identify and correct errors in a solution to a problem that involves gross or net pay.

Grade 10 Essential Mathematics – Half Course I

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:

Personal Finance (*continued*)

General Learning Outcome:

Develop an understanding of employment earnings.

Specific Learning Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

10E1.P.2.	Solve problems that require the manipulation and application of formulas related to income. [C, CN, ME, PS, R]	■ Solve a contextual problem that involves the application of a formula that does not require manipulation. ■ Solve a contextual problem that involves the application of a formula that requires manipulation, such as finding the rate of pay given regular pay and hours worked. ■ Create and solve a contextual problem that involves a formula. ■ Identify and correct errors in a solution to a problem that involves a formula.
-----------	---	---

Grade 10 Essential Mathematics – Half Course I

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Measurement		General Learning Outcome: Develop spatial sense through direct and indirect measurement.
Specific Learning Outcomes <i>It is expected that students will:</i>	Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>	
10E1.M.1. Demonstrate an understanding of the Système International (SI) by describing relationships of the units for length, area, volume, capacity, and mass. [C, CN, ME, V] It is intended that this learning outcome be limited to the base units and the prefixes milli, centi, deci, deca, hecto, and kilo.	■ Explain how the SI system was developed, and explain its relationship to base ten. ■ Identify the base units of measurement in the SI system, and determine the relationship among the related units of each type of measurement. ■ Identify contexts that involve the SI system. ■ Match the prefixes used for SI units of measurement with the powers of ten. ■ Explain, using examples, how and why decimals are used in the SI system. ■ Provide an approximate measurement in SI units for a measurement given in imperial units. ■ Convert one SI unit into another SI unit.	
10E1.M.2. Demonstrate an understanding of the imperial system by ■ describing the relationships of the units for length, area, volume, capacity, and mass ■ comparing the American and British imperial units for capacity ■ applying strategies to convert between imperial and SI units [C, CN, ME, V]	■ Explain how the imperial system was developed. ■ Identify commonly used units in the imperial system, and determine the relationships among the related units. ■ Identify contexts that involve the imperial system. ■ Explain, using examples, how and why fractions are used in the imperial system. ■ Compare the American and British imperial measurement systems, such as gallons, bushels, or tons. ■ Provide an approximate measure in imperial units for a measurement given in SI units. ■ Convert one imperial unit into another imperial unit. ■ Convert a measurement between imperial and SI units using proportional reasoning	

Grade 10 Essential Mathematics – Half course I

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Measurement (<i>continued</i>)		General Learning Outcome: Develop spatial sense through direct and indirect measurement.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
10E1.M.3.	Solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements. [CN, ME, PS, V]	■ Identify a referent for a common SI or imperial unit of linear measurement. ■ Estimate a linear measurement, using a referent. ■ Measure inside diameters, outside diameters, lengths, and widths of various given objects using various measuring instruments. ■ Measure distances using various measuring instruments. ■ Estimate the dimensions of a given regular 3-D object or 2-D shape, using a referent. ■ Determine the operations that should be used to solve a linear measurement problem. ■ Solve a linear measurement problem including perimeter, circumference, or length + width + height (used in shipping and air travel). ■ Provide an example of a situation in which a fractional linear measurement would be divided by a fraction. ■ Determine, using a variety of strategies, the midpoint of a linear measurement, such as length, width, height, depth, diagonal, or diameter of a 3-D object, and explain the strategies. ■ Explain or verify that a solution to a problem involving linear measurement is reasonable.
10E1.M.4.	Solve problems that require the manipulation and application of formulas related to converting measurement. [C, CN, ME, PS, R]	■ Solve a contextual problem that involves the application of a formula that does not require manipulation. ■ Solve a contextual problem that involves the application of a formula that requires manipulation. ■ Convert temperatures between Fahrenheit and Celsius. ■ Describe, using examples, how a given formula is used in a trade or an occupation. ■ Create and solve a contextual problem that involves a formula. ■ Identify and correct errors in a solution to a problem that involves a formula.

Grade 10 Essential Mathematics – Half Course I

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: 2-D Geometry		General Learning Outcome: Develop an understanding of spatial relationships applied to area.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
10E1.G.1.	Solve problems that involve SI and imperial area measurements of regular, composite, and irregular 2-D shapes, including decimal and fractional measurements. [ME, PS, R, V]	- Identify and compare referents for area measurements in SI and imperial units. - Estimate an area measurement, using a referent. - Identify a situation where a SI or imperial area unit would be used. - Estimate the area of a given regular, composite, or irregular 2-D shape, using a SI square grid or an imperial square grid. - Solve a contextual problem that involves the area of a regular, a composite, or an irregular 2-D shape. - Convert an area measurement expressed in one unit squared into another unit squared using proportional reasoning. - Solve a problem, using formulas for determining the areas of regular, composite, or irregular 2-D shapes, including circles. - Explain, using examples, the effect of changing the measurement of one or more dimensions on area and perimeter of rectangles. - Explain or verify that a solution to a problem involving an area measurement is reasonable.
10E1.G.2.	Solve problems that require the manipulation and application of formulas related to - perimeter - area [C, CN, ME, PS, R]	- Solve a contextual problem that involves the application of a formula that does not require manipulation. - Solve a contextual problem that involves the application of a formula that does require manipulation. - Explain or verify that different forms of the same formula are equivalent. - Describe, using examples, how a given formula is used in a trade or an occupation. - Create and solve a contextual problem that involves a formula. - Identify and correct errors in a solution to a problem that involves a formula.

Grade 10 Essential Mathematics – Half Course II

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Analysis of Games and Numbers

General Learning Outcome:
Develop critical thinking skills.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 10E2.A.1. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.
[C, CN, PS, R]

It is intended that this learning outcome be integrated throughout the course by using sliding, rotation, construction, deconstruction, and similar puzzles and games.

- Determine, explain, and verify a strategy to solve a puzzle or to win a game such as
 - guess and check
 - look for a pattern
 - make a systematic list
 - draw or model
 - eliminate possibilities
 - simplify the original problem
 - work backward
 - develop alternative approaches
- Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.
- Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Grade 10 Essential Mathematics – Half Course II

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Trigonometry		General Learning Outcome: Develop spatial sense relating to triangles.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
10E2.TG.1. Solve problems involving right triangles using the Pythagorean theorem. [C, CN, PS, V]		■ Explain, using illustrations, why the Pythagorean theorem only applies to right triangles. ■ Describe historical and contemporary applications of the Pythagorean theorem. ■ Determine if a triangle is a right triangle or if an angle is 90° using the Pythagorean theorem. ■ Solve a problem using the Pythagorean theorem.
10E2.TG.2. Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by ■ applying similarity to right triangles ■ generalizing patterns from similar right triangles ■ solving problems [CN, PS, R, T, V]		■ Explain, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side opposite to the length of the side adjacent are equal and correspond to the tangent ratio. ■ Explain, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side opposite to the length of the hypotenuse are equal and correspond to the sine ratio. ■ Explain, for a specified acute angle in a set of similar right triangles, that the ratios of the length of the side adjacent to the length of the hypotenuse are equal and correspond to the cosine ratio. ■ Identify situations where the trigonometric ratios are used for indirect measurement of angles and lengths. ■ Solve a contextual problem that involves right triangles, using the primary trigonometric ratios. ■ Explain or verify that a solution to a problem involving primary trigonometric ratios is reasonable.

Grade 10 Essential Mathematics – Half Course II

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:

Trigonometry (*continued*)

General Learning Outcome:

Develop spatial sense relating to triangles.

Specific Learning Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 10E2.TG.3. Solve problems that require the manipulation and application of formulas related to
- the Pythagorean theorem
 - primary trigonometric ratios
- [C, CN, ME, PS, R]

- Solve a contextual problem that involves the application of a formula that does not require manipulation.
- Solve a contextual problem that involves the application of a formula that does require manipulation.
- Explain or verify that different forms of the same formula are equivalent.
- Describe, using examples, how a given formula is used in a trade or an occupation.
- Create and solve a contextual problem that involves a formula.
- Identify and correct errors in a solution to a problem that involves a formula.

Grade 10 Essential Mathematics – Half Course II

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Consumer Decisions		General Learning Outcome: Develop skills to make informed consumer decisions.
Specific Learning Outcomes <i>It is expected that students will:</i>	Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>	
10E2.C.1. Solve problems that involve unit pricing and currency exchange, using proportional reasoning. [CN, ME, PS, R]	■ Determine or compare the unit price of two or more items. ■ Solve problems that involve determining the best buy, and explain the choice in terms of the cost as well as other factors, such as quality and quantity. ■ Compare, using examples, different sales promotion techniques ■ Determine the sale price of an item when given the percent of discount. ■ Determine the percent increase or decrease of an item. ■ Solve, using proportional reasoning, a contextual problem that involves currency exchange. ■ Explain the difference between the selling rate and purchasing rate for currency exchange. ■ Explain how to estimate the cost of items in Canadian currency while in a foreign country, and explain why this may be important. ■ Convert between Canadian currency and foreign currencies, using formulas, charts, or tables.	

Grade 10 Essential Mathematics – Half Course II

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Transformations

General Learning Outcome:
Develop spatial sense.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 10E2.TF.1. Demonstrate an understanding of transformations on a 2-D shape, including
- translations
 - rotations
 - reflections
 - dilations
- [C, CN, R, T, V]

- Identify a single transformation that was performed, given the original 2-D shape and its image.
- Draw the image of a 2-D shape that results from a single transformation.
- Draw the image of a 2-D shape that results from a combination of successive transformations.
- Create, analyze, and describe designs, using translations, rotations, and reflections .
- Identify and describe applications of transformations in construction, industrial, commercial, domestic, and artistic contexts.
- Explain the relationship between reflections and lines or planes of symmetry.
- Determine and explain whether an image is a dilation of another shape, using the concept of similarity.
- Draw, with or without technology, a dilation image for a 2-D shape, and explain how the original 2-D shape and its image are proportional.
- Solve a contextual problem that involves transformations.

Grade 10 Essential Mathematics – Half Course II

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Angle Construction		General Learning Outcome: Develop spatial sense.
Specific Learning Outcomes <i>It is expected that students will:</i>	Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>	
10E2.AC.1. Demonstrate an understanding of angles, including acute, right, obtuse, straight, and reflex, by ■ drawing ■ replicating and constructing ■ bisecting ■ solving problems [C, ME, PS, T, V]	■ Draw and describe angles with various measures, including acute, right, straight, obtuse, or reflex angles. ■ Identify referents for angles. ■ Sketch a given angle. ■ Estimate the measure of a given angle, using 22.5°, 30°, 45°, 60°, 90°, and 180° as referent angles. ■ Measure, using a protractor, angles in various orientations. ■ Explain and illustrate how angles can be replicated in a variety of ways, such as with Mira, protractor, compass and straightedge, carpenter's square, or dynamic geometry software. ■ Replicate angles in a variety of ways, with or without technology. ■ Bisect an angle, using a variety of methods. ■ Identify and describe the use of angles in construction, industrial, commercial, or artistic applications. ■ Solve a contextual problem that involves angles.	
10E2.AC.2. Solve problems that involve parallel, perpendicular, and transversal lines, and pairs of angles formed between them. [C, CN, PS, V]	■ Identify a set of lines as perpendicular, parallel, or neither, and explain. ■ Illustrate and describe complementary or supplementary angles. ■ Identify, in a set of angles, adjacent angles that are not complementary or supplementary. ■ Explain and illustrate the relationships of angles formed by parallel lines and a transversal. ■ Determine the measures of angles involving parallel lines and a transversal, using angle relationships. ■ Explain, using examples, why the angle relationships do not apply when the lines are not parallel. ■ Determine if lines or planes are perpendicular or parallel, and describe the strategy used. ■ Identify and describe the use of parallel, perpendicular, and transversal lines in construction, industrial, commercial, or artistic applications. ■ Solve a contextual problem that involves angles formed by parallel lines and a transversal.	

GRADE 10 INTRODUCTION TO APPLIED AND PRE-CALCULUS MATHEMATICS (20S)

Grade 10 Introduction to Applied and Pre-Calculus Mathematics (20S) is intended for students considering post-secondary studies that require a math prerequisite. This pathway provides students with the mathematical understanding and critical-thinking skills that have been identified for specific post-secondary programs of study. The topics studied form the foundation for topics to be studied in both Grade 11 Applied Mathematics and Grade 11 Pre-Calculus Mathematics.

Components of the curriculum are both context-driven and algebraic in nature. Students will engage in experiments and activities that include the use of technology, problem solving, mental mathematics, and theoretical mathematics to promote the development of mathematical skills. These experiences will provide opportunities for students to make connections between symbolic mathematical ideas and the world around us.

Assessment of Grade 10 Introduction to Applied and Pre-Calculus Mathematics should be a balance of assessment *for* learning, assessment *as* learning, and assessment *of* learning. Assessment tools used in Grade 10 Introduction to Applied and Pre-Calculus Mathematics should be varied and may include observation, homework, learning conversations or interviews, summative unit essays, demonstrations, presentations, performance

tasks, learning logs, projects, investigations, reflective journals, portfolios, quizzes, tests, and examinations. An appropriately prepared portfolio requires a consistent effort throughout the school term and a commitment to completing quality work on a daily basis.

The learning outcomes are divided into three topics: Measurement; Algebra and Number; and Relations and Functions. For instructional purposes, the learning outcomes could be arranged into units. Learning outcomes from different topics could be taught in the same unit. Some learning outcomes may fit into multiple units, and parts of the outcome could be taught in one unit while the remaining parts can be taught later. Two possible sequences of the learning outcomes into units with suggested time allotments follow. These are not the only possibilities, but they will provide some direction for teachers for their first time through the course. The suggested time includes instructional and assessment time.

Regardless of the organization of the learning outcomes into units, students should constantly be looking for and be given opportunities to see connections between the various learning outcomes in Grade 10 Introduction to Applied and Pre-Calculus Mathematics.

Possibility 1			Possibility 2		
Unit	Learning Outcomes	Suggested Hours	Unit	Learning Outcomes	Suggested Hours
Linear Modelling	R1, R4, R5	15	Graphs and Relations	R1, R3, R4, R5	20
Number Sense	A1, A2	5	Number Sense	A1, A2, A3	10
Measurement	M1, M2, M3	20	Linear Measurement	M1, M2	15
Linear Functions	R2, R8	10	Trigonometry	M4	10
Algebra	A3, A4, A5	15	Relations and Functions	R2, R8	10
Coordinate Geometry	R3, R6, R10	15	Polynomials	A3, A4, A5	15
Trigonometry	M4	10	Coordinate Geometry	R3, R5, R6, R7, R10	20
Applications of Linear Functions	R1, R7, R9	20	Surface Area and Volume	M3	5
			Systems of Equations	R9	5
Total		110	Total		110

General and Specific Learning Outcomes with Achievement Indicators by Course

Grade 10 – Introduction to Applied and Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Measurement		General Learning Outcome: Develop spatial sense and proportional reasoning.	
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>	
10I.M.1.	Solve problems that involve linear measurement, using ■ SI and imperial units of measure ■ estimation strategies ■ measurement strategies [ME, PS, V]	■ Provide referents for linear measurements, including millimetre, centimetre, metre, kilometre, inch, foot, yard, and mile, and explain the choices. ■ Compare SI and imperial units, using referents. ■ Estimate a linear measure, using a referent, and explain the process used. ■ Justify the choice of units used for determining a measurement in a problem-solving context. ■ Solve a contextual problem that involves linear measure, using instruments such as rulers, tape measures, trundle wheels, micrometers, or calipers. ■ Explain a personal strategy used to determine a linear measurement such as the circumference of a bottle, the length of a curve, or the perimeter of the base of an irregular 3-D object, and explain why it works.	
10I.M.2.	Apply proportional reasoning to problems that involve conversions within and between SI and imperial units of measure. [C, ME, PS, T]	■ Explain how proportional reasoning can be used to convert a measurement within or between SI and imperial systems. ■ Solve a contextual problem that involves the conversion of units within or between SI and imperial systems. ■ Justify, using mental mathematics, the reasonableness of a solution to a conversion problem.	

Grade 10 – Introduction to Applied and Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Measurement (<i>continued</i>)		General Learning Outcome: Develop spatial sense and proportional reasoning.	
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>	
10I.M.3.	Solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including ■ right cones ■ right cylinders ■ right prisms ■ right pyramids ■ spheres [CN, PS, R, T, V]	■ Sketch a diagram to represent a problem that involves surface area or volume. ■ Determine the surface area (may be expressed with scientific notation) of a right cone, right cylinder, right prism, right pyramid, or sphere, using an object or its labelled diagram. ■ Determine the volume (may be expressed with scientific notation) of a right cone, right cylinder, right prism, right pyramid, or sphere, using an object or its labelled diagram. ■ Determine an unknown dimension of a right cone, right cylinder, right prism, right pyramid, or sphere, given the object's surface area or volume and the remaining dimensions. ■ Solve a contextual problem that involves surface area or volume, given a diagram of a composite 3-D object. ■ Describe the relationship between the volumes of ■ right cones and right cylinders with the same base and height ■ right pyramids and right prisms with the same base and height	
10I.M.4.	Develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles. [C, CN, PS, R, T, V]	■ Explain the relationships between similar right triangles and the definitions of the primary trigonometric ratios. ■ Identify the hypotenuse of a right triangle and the opposite and adjacent sides for a given acute angle in the triangle. ■ Solve a problem that involves one or more right triangles by applying the primary trigonometric ratios or the Pythagorean theorem. ■ Solve a problem that involves direct and indirect measurement, using measurement instruments such as a clinometer or metre stick, the trigonometric ratios, or the Pythagorean theorem.	

Grade 10 – Introduction to Applied and Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Algebra and Number

General Learning Outcome:
Develop algebraic reasoning and number sense.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

10I.A.1.	Demonstrate an understanding of factors of whole numbers by determining ■ prime factors ■ greatest common factor ■ least common multiple ■ square root ■ cube root [CN, ME, R]	■ Determine the prime factors of a whole number. ■ Explain why the numbers 0 and 1 are special cases when doing prime factorization. ■ Determine, using a variety of strategies, the greatest common factor or least common multiple of a set of whole numbers, and explain the process. ■ Determine, concretely or pictorially, whether a whole number is a perfect square, a perfect cube, or neither. ■ Determine, using a variety of strategies, the square root of a perfect square, and explain the process. ■ Determine, using a variety of strategies, the cube root of a perfect cube, and explain the process. ■ Solve a problem that involves prime factors, greatest common factors, least common multiples, square roots, or cube roots.
10I.A.2.	Demonstrate an understanding of irrational numbers by ■ representing, identifying, and simplifying irrational numbers ■ ordering irrational numbers [CN, ME, R, V]	■ Sort a set of numbers into rational and irrational numbers. ■ Determine an approximate value of an irrational number. ■ Approximate the locations of irrational numbers on a horizontal or vertical number line, using a variety of strategies, and explain the reasoning. ■ Order a set of irrational numbers on a horizontal or vertical number line. ■ Express a radical as a mixed radical in simplest form (limited to numerical radicands). ■ Express a mixed radical as an entire radical (limited to numerical radicands). ■ Explain, using examples, the meaning of the index of a radical. ■ Represent, using a graphic organizer, the relationship among the subsets of the real numbers (natural, whole, integer, rational, irrational).

Grade 10 – Introduction to Applied and Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Algebra and Number (*continued*)

General Learning Outcome:
Develop algebraic reasoning and number sense.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 10I.A.3. Demonstrate an understanding of powers with integral and rational exponents.
[C, CN, PS, R]

This learning outcome offers the opportunity to expose students to scientific notation.

- Explain, using patterns or exponent laws, why $x^{-n} = \frac{1}{x^n}$, $x \neq 0$
- Explain, using patterns, why $x^{\frac{1}{n}} = \sqrt[n]{x}$, $n \neq 0$
- Apply the exponent laws to expressions with rational or variable bases and integral or rational exponents, and explain the reasoning
 - $(x^m)(x^n) = x^{m+n}$
 - $x^m \div x^n = x^{m-n}$, $x \neq 0$
 - $(x^m)^n = x^{mn}$
 - $(xy)^m = x^m y^m$
 - $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$, $y \neq 0$
- Express powers with rational exponents as radicals and vice versa.
- Solve a problem that involves exponent laws or radicals.
- Identify and correct errors in the simplification of an expression that involves powers.

Grade 10 – Introduction to Applied and Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Algebra and Number (<i>continued</i>)		General Learning Outcome: Develop algebraic reasoning and number sense.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
10I.A.4.	Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials), concretely, pictorially, and symbolically. [C, CN, R, V] It is intended that the emphasis of this learning outcome be on binomial-by-binomial multiplication, with extension to polynomial-by-polynomial to establish a general pattern for multiplication.	■ Model the multiplication of two binomials, concretely or pictorially, and record the process symbolically. ■ Relate the multiplication of two binomial expressions to an area model. ■ Explain, using examples, the relationship between the multiplication of binomials and the multiplication of two-digit numbers. ■ Verify a polynomial product by substituting numbers for the variables. ■ Multiply two polynomials symbolically, and combine like terms in the product. ■ Generalize and explain a strategy for multiplication of polynomials. ■ Identify and explain errors in a solution for a polynomial multiplication.
10I.A.5.	Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially, and symbolically. [C, CN, R, V]	■ Determine the common factors in the terms of a polynomial, and express the polynomial in factored form. ■ Model the factoring of a trinomial, concretely or pictorially, and record the process symbolically. ■ Factor a polynomial that is a difference of squares, and explain why it is a special case of factoring a trinomial of the form $ax^2 + bx + c = 0$ where $b = 0$ and $c < 0$. ■ Identify and explain errors in a polynomial factorization. ■ Factor a polynomial, and verify by multiplying the factors. ■ Explain, using examples, the relationship between multiplication and factoring of polynomials. ■ Generalize and explain strategies used to factor a trinomial. ■ Express a polynomial as a product of its factors.

Grade 10 – Introduction to Applied and Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions

General Learning Outcome:
Develop algebraic and graphical reasoning through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

10I.R.1.	Interpret and explain the relationships among data, graphs, and contexts. [C, CN, R, T, V]	■ Graph, with or without technology, a set of data, and determine the restrictions on the domain and range. ■ Explain why data points should or should not be connected on the graph for a context. ■ Match corresponding representations of data, graphs, and contexts. ■ Describe a possible context for a given graph. ■ Sketch a possible graph for a context. ■ Describe the restrictions on the domain and range for a context.
10I.R.2.	Demonstrate an understanding of relations and functions. [C, R, V]	■ Explain, using examples, why some relations are not functions but all functions are relations. ■ Determine whether a set of ordered pairs represents a function. ■ Sort a set of graphs as functions or non-functions. ■ Generalize and explain rules for determining whether graphs and sets of ordered pairs represent functions. ■ Determine and express in a variety of ways the domain and range of a relation.
10I.R.3.	Demonstrate an understanding of slope with respect to ■ rise and run ■ line segments and lines ■ rate of change ■ parallel lines ■ perpendicular lines [PS, R, V]	■ Explain, using examples, slope as a rate of change. ■ Determine the slope of a line segment by measuring or calculating the rise and run. ■ Classify lines in a set as having positive or negative slopes. ■ Explain the meaning of the slope of a horizontal or vertical line. ■ Explain why the slope of a line can be determined by using any two points on that line. ■ Draw a line, given its slope and a point on the line. ■ Determine another point on a line, given the slope and a point on the line. ■ Generalize and apply a rule for determining whether two lines are parallel or perpendicular. ■ Solve a contextual problem involving slope.

Grade 10 – Introduction to Applied and Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

10I.R.4.	Describe and represent linear relations, using ■ words ■ ordered pairs ■ tables of values ■ graphs ■ equations [C, CN, R, V]	■ Identify independent and dependent variables in a context. ■ Determine and explain whether a graph represents a linear relation. ■ Determine and explain whether a context represents a linear relation. ■ Determine and explain whether a table of values or a set of ordered pairs represents a linear relation. ■ Draw a graph from a set of ordered pairs within a context, and determine whether the relationship between the variables is linear. ■ Determine and explain whether an equation represents a linear relation. ■ Match corresponding representations of linear relations.
10I.R.5.	Determine the characteristics of the graphs of linear relations, including the ■ intercepts ■ slope ■ domain ■ range [CN, PS, R, T, V]	■ Determine the intercepts of the graph of a linear relation, and state the intercepts as values or ordered pairs. ■ Determine the slope of the graph of a linear relation. ■ Determine the domain and range of the graph of a linear relation. ■ Sketch a linear relation that has one, two, or an infinite number of intercepts. ■ Match graphs to their corresponding slopes and y-intercepts. ■ Solve a contextual problem that involves intercepts, slope, domain, or range of a linear relation.

Grade 10 – Introduction to Applied and Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

10I.R.6.	Relate linear relations expressed in ■ slope–intercept form ($y = mx + b$) ■ general form ($Ax + By + C = 0$) ■ slope–point form ($y - y_1 = m(x - x_1)$) to their graphs. [C, CN, R, T, V]	■ Express a linear relation in different forms. ■ Generalize and explain strategies for graphing a linear relation in slope–intercept, general, or slope–point form. ■ Graph, with or without technology, a linear relation given in slope–intercept, general, or slope–point form, and explain the strategy used to create the graph. ■ Identify equivalent linear relations from a set of linear relations. ■ Match a set of linear relations to their graphs.
10I.R.7.	Determine the equation of a linear relation, given ■ a graph ■ a point and the slope ■ two points ■ a point and the equation of a parallel or perpendicular line ■ a scatterplot [C, CN, PS, R, T, V]	■ Determine the slope and y-intercept of a linear relation from its graph, and write the equation in the form $y = mx + b$. ■ Write the equation of a linear relation, given its slope and the coordinates of a point on the line, and explain the process. ■ Write the equation of a linear relation, given the coordinates of two points on the line, and explain the process. ■ Write the equation of a linear relation, given the coordinates of a point on the line and the equation of a parallel or perpendicular line, and explain the process. ■ Graph linear data generated from a context, and write the equation of the resulting line. ■ Determine the equation of the line of best fit from a scatterplot using technology, and discuss the correlation. ■ Solve a contextual problem, using the equation of a linear relation.

Grade 10 – Introduction to Applied and Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

10I.R.8.	Represent a linear function, using function notation. [CN, ME, V]	■ Express the equation of a linear function in two variables, using function notation. ■ Express an equation given in function notation as a linear function in two variables. ■ Determine the related range value, given a domain value for a linear function. ■ Determine the related domain value, given a range value for a linear function. ■ Sketch the graph of a linear function expressed in function notation.
10I.R.9.	Solve problems that involve systems of linear equations in two variables, graphically and algebraically. [CN, PS, R, T, V]	■ Model a situation, using a system of linear equations. ■ Relate a system of linear equations to the context of a problem. ■ Determine and verify the solution of a system of linear equations graphically, with or without technology. ■ Explain the meaning of the point of intersection of a system of linear equations. ■ Determine and verify the solution of a system of linear equations algebraically. ■ Explain, using examples, why a system of equations may have no solution, one solution, or an infinite number of solutions. ■ Describe a strategy to solve a system of linear equations. ■ Solve a contextual problem that involves a system of linear equations, with or without technology.
10I.R.10.	Solve problems that involve the distance between two points and the midpoint of a line segment. [C, CN, PS, T, V]	■ Determine the distance between two points on a Cartesian plane, using a variety of strategies. ■ Determine the midpoint of a line segment, given the endpoints of the segment, using a variety of strategies. ■ Determine an endpoint of a line segment, given the other endpoint and the midpoint, using a variety of strategies. ■ Solve a contextual problem involving distance between two points or midpoint of a line segment.

GRADE 11 APPLIED MATHEMATICS (30S)

Grade 11 Applied Mathematics (30S) is intended for students considering post-secondary studies who do not require a study of theoretical calculus. It is context-driven and promotes the learning of numerical and geometrical problem-solving techniques as they relate to the world around us. It builds upon the foundation knowledge and skills from Grade 10 Introduction to Applied and Pre-Calculus Mathematics and builds a foundation for Grade 12 Applied Mathematics.

Primary goals of Applied Mathematics are to have students develop critical-thinking skills through problem solving and model real-world situations mathematically to make predictions.

These goals may be attained in a number of ways. Students may collect data in experiments and activities and then develop mathematical concepts by analyzing that data. They are encouraged to learn and demonstrate effective communication skills through a variety of media. Students are expected to become proficient in both oral and written communication skills.

Applied Mathematics is designed to promote student flexibility and responsibility. Flexibility is encouraged by having students work on non-routine problems and projects. Responsibility is encouraged as students work individually and in cooperative groups to explore connections with other mathematical areas, school subjects, and real-life applications.

Technology is an integral part of both learning and assessment in Applied Mathematics. Graphing calculators, spreadsheets, or other computer software will be used by students for mathematical explorations, modelling, and problem solving.

Assessment of Grade 11 Applied Mathematics should be a balance of assessment *for* learning, assessment *as* learning, and assessment *of* learning. Assessment tools used in Grade 11 Applied Mathematics should be varied and may include observation, homework, learning conversations or interviews, summative unit essays, demonstrations, presentations, performance tasks, learning logs, projects, investigations, reflective journals, portfolios, quizzes, tests, and examinations. An appropriately prepared portfolio requires a consistent effort throughout the school term and a commitment to completing quality work on a daily basis.

The Grade 11 Applied Mathematics course includes the following topics: measurement, geometry, logical reasoning, statistics, and relations and functions. Additionally, students will complete a Mathematics Research Project.

Learning outcomes from various topic areas may be combined in any order when considering learning activities. Two possible teaching sequences and suggested hours are presented in the following tables. The time suggested includes time for instructional and assessment time.

Possibility 1			Possibility 2		
Unit	Learning Outcomes	Suggested Hours	Unit	Learning Outcomes	Suggested Hours
Quadratic Functions	R2, M1, L2	20	Problem Solving	M1, L2	5
Scale	M2, M3, M1, L2	15	Quadratic Functions	R2	15
Proofs	G1, G2, L1, L2	10	Research Project	RP1	10
Statistics	S1, S2, L2	20	Proofs	G1, G2, L1, L2	10
Research Project	RP1, L2	10	Statistics	S1, S2, L2	20
Systems of Inequalities	R1, M1, L2	20	Systems of Inequalities	R1, L2	20
Trigonometry	G3, G2, L1, L2	15	Trigonometry	G3, L1, L2	15
			Scale	M2, M3, L2	15
Total		110	Total		110

General and Specific Learning Outcomes with Achievement Indicators by Course

Grade 11 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Measurement		General Learning Outcome: Develop spatial sense and proportional reasoning.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
11A.M.1.	Solve problems that involve the application of rates. [CN, PS, R, T]	■ Interpret rates in a context, such as the arts, commerce, the environment, medicine, or recreation. ■ Solve a rate problem that requires the isolation of a variable. ■ Determine and compare rates and unit rates. ■ Make and justify a decision, using rates. ■ Interpret a graph that represents a rate. ■ Draw a graph to represent a rate. ■ Explain, using examples, the relationship between the slope of a graph and a rate. ■ Describe a context for a rate or unit rate. ■ Identify and explain factors that influence a rate in a context. ■ Solve a contextual problem that involves rates or unit rates.

Grade 11 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Measurement (<i>continued</i>)		General Learning Outcome: Develop spatial sense and proportional reasoning.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
11A.M.2.	Solve problems that involve scale diagrams, using proportional reasoning. [CN, PS, R, T, V]	■ Provide examples where scale diagrams are used to model a 2-D shape or a 3-D object. ■ Determine, using proportional reasoning, the scale factor, given one dimension of a 2-D shape or a 3-D object and its representation. ■ Determine, using proportional reasoning, an unknown dimension of a 2-D shape or a 3-D object, given a scale diagram or a model. ■ Draw, with or without technology, a scale diagram of a 2-D shape according to a specified scale factor (enlargement or reduction). ■ Solve a contextual problem that involves scale diagrams.
11A.M.3.	Demonstrate an understanding of the relationships among scale factors, areas, surface areas, and volumes of similar 2-D shapes and 3-D objects. [C, CN, PS, R, T, V]	■ Determine the area of a 2-D shape, given the scale diagram, and justify the reasonableness of the result. ■ Determine the surface area and volume of a 3-D object, given the scale diagram, and justify the reasonableness of the result. ■ Explain, using examples, the effect of a change in the scale factor on the area of a 2-D shape. ■ Explain, using examples, the effect of a change in the scale factor on the surface area of a 3-D object. ■ Explain, using examples, the effect of a change in the scale factor on the volume of a 3-D object. ■ Explain, using examples, the relationships among scale factor, area of a 2-D shape, surface area of a 3-D object, and volume of a 3-D object. ■ Solve a spatial problem that requires the manipulation of formulas. ■ Solve a contextual problem that involves the relationships among scale factors, areas, and volumes.

Grade 11 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Geometry

General Learning Outcome:
Develop spatial sense.

Specific Learning Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11A.G.1.	Derive proofs that involve the properties of angles and triangles. [CN, R, T, V]	It is intended that deductive reasoning be limited to direct proof. ■ Generalize, using inductive reasoning, the relationships between pairs of angles formed by transversals and parallel lines, with or without technology. ■ Prove, using deductive reasoning, properties of angles formed by transversals and parallel lines, including the sum of the angles in a triangle. ■ Generalize, using inductive reasoning, a rule for the relationship between the sum of the interior angles and the number of sides (n) in a polygon, with or without technology. ■ Identify and correct errors in a proof of a property involving angles. ■ Verify, with examples, that if lines are not parallel, the angle properties do not apply.
11A.G.2.	Solve problems that involve the properties of angles and triangles. [CN, PS, T, V]	■ Determine the measures of angles in a diagram that involves parallel lines, angles, and triangles, and justify the reasoning. ■ Identify and correct errors in a solution to a problem that involves the measure of angles. ■ Solve a contextual problem that involves angles or triangles. ■ Construct parallel lines, using only a compass or a protractor, and explain the strategy used. ■ Determine whether lines are parallel, given the measure of an angle at each intersection formed by the lines and a transversal.
11A.G.3.	Solve problems that involve the cosine law and the sine law, including the ambiguous case. [CN, PS, R, T]	■ Sketch a diagram to represent a problem that involves the cosine law or sine law. ■ Explain the steps in a proof of the sine law or cosine law. ■ Solve a problem involving the cosine law that requires the manipulation of a formula. ■ Explain, concretely, pictorially, or symbolically, whether zero, one, or two triangles exist, given two sides and a non-included angle. ■ Solve a problem involving the sine law that requires the manipulation of a formula. ■ Solve a contextual problem that involves the cosine law or the sine law.

Grade 11 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Logical Reasoning

General Learning Outcome:
Develop logical reasoning.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 11A.L.1. Analyze and prove conjectures, using inductive and deductive reasoning, to solve problems.
[C, CN, PS, R, T]

It is intended that this learning outcome be integrated throughout the course.

- Make conjectures by observing patterns and identifying properties, and justify the reasoning.
- Explain why inductive reasoning may lead to a false conjecture.
- Compare, using examples, inductive and deductive reasoning.
- Provide and explain a counter-example to disprove a conjecture.
- Prove algebraic and number relationships, such as divisibility rules, number properties, mental mathematics strategies, or algebraic number tricks.
- Prove a conjecture, using deductive reasoning (not limited to two column proofs).
- Determine whether an argument is valid, and justify the reasoning.
- Identify errors in a proof.
- Solve a contextual problem involving inductive or deductive reasoning.

- 11A.L.2. Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.
[CN, PS, R, T, V]

It is intended that this learning outcome be integrated throughout the course by using sliding, rotation, construction, deconstruction, and similar puzzles and games.

- Determine, explain, and verify a strategy to solve a puzzle or to win a game such as
 - guess and check
 - look for a pattern
 - make a systematic list
 - draw or model
 - eliminate possibilities
 - simplify the original problem
 - work backward
 - develop alternative approaches
- Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.
- Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Grade 11 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Statistics

General Learning Outcome:
Develop statistical reasoning.

Specific Learning Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 11A.S.1. Demonstrate an understanding of normal distribution, including
- standard deviation
 - z-scores
- [CN, PS, T, V]

- Explain, using examples, the meaning of standard deviation.
- Calculate, using technology, the population standard deviation of a data set.
- Explain, using examples, the properties of a normal curve, including the mean, median, mode, standard deviation, symmetry, and area under the curve.
- Determine whether a data set approximates a normal distribution, and explain the reasoning.
- Compare the properties of two or more normally distributed data sets.
- Explain, using examples that represent multiple perspectives, the application of standard deviation for making decisions in situations, such as warranties, insurance, or opinion polls.
- Solve a contextual problem that involves the interpretation of standard deviation.
- Determine, with or without technology, and explain the z-score for a value in a normally distributed data set.
- Solve a contextual problem that involves normal distribution.

Grade 11 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Statistics (*continued*)

General Learning Outcome:
Develop statistical reasoning.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11A.S.2. Interpret statistical data, using

- confidence intervals
- confidence levels
- margin of error

[C, CN, R, T]

It is intended that the focus of this learning outcome be on interpretation of data rather than on statistical calculations.

- Explain, using examples, how confidence levels, margin of error, and confidence intervals may vary depending on the size of the random sample.
- Explain, using examples, the significance of a confidence interval, margin of error, or confidence level.
- Make inferences about a population from sample data, using confidence intervals, and explain the reasoning.
- Provide examples from print or electronic media in which confidence intervals and confidence levels are used to support a particular position.
- Interpret and explain confidence intervals and margin of error, using examples found in print or electronic media.
- Support a position by analyzing statistical data presented in the media.

Grade 11 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions

General Learning Outcome:
Develop algebraic and graphical reasoning
through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11A.R.1. Model and solve problems that involve systems of linear inequalities in two variables.
[CN, PS, T, V]

- Model a problem, using a system of linear inequalities in two variables.
- Graph the boundary line between two half planes for each inequality in a system of linear inequalities, and justify the choice of solid or broken lines.
- Determine and explain the solution region that satisfies a linear inequality, using a variety of strategies when given a boundary line.
- Determine, graphically, the solution region for a system of linear inequalities, and verify the solution.
- Explain, using examples, the significance of the shaded region in the graphical solution of a system of linear inequalities.
- Solve an optimization problem, using linear programming.

Grade 11 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 11A.R.2. Demonstrate an understanding of the characteristics of quadratic functions, including
- vertex
 - intercepts
 - domain and range
 - axis of symmetry
- [CN, PS, T, V]

It is intended that completion of the square not be required.

- Determine, with technology, the intercepts of the graph of a quadratic function or the roots of the corresponding quadratic equation.
- Explain the relationships among the roots of an equation, the zeros of the corresponding function, and the x-intercepts of the graph of the function.
- Explain, using examples, why the graph of a quadratic function may have zero, one, or two x intercepts.
- Express a quadratic equation in factored form, using the zeros of a corresponding function or the x-intercepts of its graph.
- Determine, with technology, the coordinates of the vertex of the graph of a quadratic function.
- Determine the equation of the axis of symmetry of the graph of a quadratic function, given the x-intercepts of the graph.
- Determine the coordinates of the vertex of the graph of a quadratic function, given the equation of the function and the axis of symmetry, and determine whether the y-coordinate of the vertex is a maximum or a minimum.
- Determine the domain and range of a quadratic function.
- Sketch the graph of a quadratic function.
- Solve, with technology, a contextual problem involving data that is best represented by graphs of quadratic functions, and explain the reasoning.
- Solve a contextual problem that involves the characteristics of a quadratic function.

Grade 11 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Mathematics Research Project		General Learning Outcome: Develop an appreciation of the role of mathematics in society.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
11A.RP.1.	Research and give a presentation on a historical event or an area of interest that involves mathematics. [C, CN, ME, PS, R, T, V]	■ Collect primary or secondary data (statistical or informational) related to the topic. ■ Assess the accuracy, reliability, and relevance of the primary or secondary data collected by ■ identifying examples of bias and points of view ■ identifying and describing the data collection methods ■ determining whether the data is relevant ■ determining whether the data is consistent with information obtained from other sources on the same topic ■ Interpret data, using statistical methods if applicable. ■ Identify controversial issues, if any, and present multiple sides of the issues with supporting data. ■ Organize and present the research project, with or without technology.

GRADE 11 ESSENTIAL MATHEMATICS (30S)

Grade 11 Essential Mathematics (30S) is intended for students whose post-secondary planning does not include a focus on mathematics and science-related fields. Grade 11 Essential Mathematics is a one-credit course consisting of two half-credits each emphasizing consumer applications, problem solving, decision making, and spatial sense. Grade 11 Essential Mathematics builds on the knowledge and skills of Grade 10 Essential Mathematics and provides a foundation for the topics studied in Grade 12 Essential Mathematics.

Students are expected to work both individually and in small groups on mathematical concepts and skills encountered in everyday life in a technological society.

Assessment of Grade 11 Essential Mathematics should be a balance of assessment *for* learning, assessment *as* learning,

and assessment *of* learning. Assessment tools used in Grade 11 Essential Mathematics should be varied and may include observation, homework, learning conversations or interviews, summative unit essays, demonstrations, presentations, performance tasks, learning logs, projects, investigations, reflective journals, portfolios, quizzes, tests, and examinations. An appropriately prepared portfolio requires a consistent effort throughout the school term and a commitment to completing quality work on a daily basis.

The following table lists the units of study for each half-course along with an estimated number of hours for each unit. The time for each unit includes instructional and assessment time.

Grade 11 Essential Mathematics (35S) Half Course III		Grade 11 Essential Mathematics (35S) Half Course IV	
Unit	Suggested Hours	Unit	Suggested Hours
Analysis of Games and Numbers	7	Analysis of Games and Numbers	5
Interest and Credit	18	Managing Money	16
3-D Geometry	16	Relations and Patterns	16
Statistics	14	Trigonometry	5
		Design Modelling	13
Total	55	Total	55

General and Specific Learning Outcomes with Achievement Indicators by Course

Grade 11 Essential Mathematics – Half Course III

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Analysis of Games and Numbers		General Learning Outcome: Develop critical thinking skills.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
11E3.A.1.	Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]	■ Determine, explain, and verify a strategy to solve a puzzle or to win a game such as ■ guess and check ■ look for a pattern ■ make a systematic list ■ draw or model ■ eliminate possibilities ■ simplify the original problem ■ work backward ■ develop alternative approaches ■ Identify and correct errors in a solution to a puzzle or in a strategy for winning a game. ■ Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

It is intended that this learning outcome be integrated throughout the course by using puzzles and games such as cribbage, magic squares, and Kakuro.

Grade 11 Essential Mathematics – Half Course III

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Interest and Credit		General Learning Outcome: Develop an understanding of credit and the effects of interest.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
11E3.I.1.	Demonstrate an understanding of compound interest. [CN, ME, PS, T]	■ Solve a problem that involves simple interest, given three of the four values in the formula $I = Prt$. ■ Compare simple and compound interest, and explain their relationship. ■ Solve, using a formula, a contextual problem that involves compound interest. ■ Solve “what if” questions involving compound interest using technology. ■ Explain, using examples, the effect of different compounding periods on calculations of compound interest. ■ Estimate, using the Rule of 72, the time required for a given investment to double in value. ■ Compare the advantages and disadvantages of different types of investment options.
11E3.I.2.	Demonstrate an understanding of credit options, including ■ credit cards ■ loans [CN, ME, PS, R]	■ Compare advantages and disadvantages of different types of credit options, including bank or store credit cards, personal loans, lines of credit, overdraft. ■ Make informed decisions related to the use of credit, such as service charges, interest, payday loans, or sales promotions, and explain the reasoning. ■ Describe strategies to use credit effectively, such as negotiating interest rates, planning payment timelines, reducing accumulated debt, or timing purchases. ■ Compare credit card options from various companies and financial institutions. ■ Solve a contextual problem that involves credit cards or loans. ■ Solve a contextual problem that involves credit linked to sales promotions.
11E3.I.3.	Solve problems that require the manipulation and application of formulas related to ■ simple interest ■ finance charges [CN, PS, R]	■ Solve a contextual problem involving the application of a formula that does not require manipulation. ■ Solve a contextual problem involving the application of a formula that requires manipulation. ■ Explain or verify that different forms of the same formula are equivalent. ■ Describe, using examples, how a given formula is used in a trade or an occupation. ■ Create and solve a contextual problem that involves a formula. ■ Identify and correct errors in a solution to a problem that involves a formula.

Grade 11 Essential Mathematics – Half Course III

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
3-D Geometry

General Learning Outcome:
Develop an understanding of spatial relationships
applied to volume and surface area.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11E3.G.1. Solve problems that involve SI and imperial units in surface area measurements.

[C, CN, ME, PS, V]

It is intended that arithmetic operations on decimals and fractions be integrated into the problems.

- Explain, using examples, the difference between volume and surface area.
- Explain, using examples, including nets, the relationship between area and surface area.
- Explain how a referent can be used to estimate surface area.
- Estimate the surface area of a 3-D object.
- Illustrate, using examples, the effect of dimensional changes on surface area.
- Solve a contextual problem that involves the surface area of 3-D objects, including spheres, and that requires the manipulation of formulas.

Grade 11 Essential Mathematics –Half Course III

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:

3-D Geometry (*continued*)

General Learning Outcome:

Develop an understanding of spatial relationships applied to volume and surface area.

Specific Learning Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 11E3.G.2. Solve problems that involve SI and imperial units in volume and capacity measurements.

[C, CN, ME, PS, V]

It is intended that arithmetic operations on decimals and fractions be integrated into the problems.

- Explain, using examples, the difference between volume and capacity.
- Identify and compare referents for volume and capacity measurements in SI and imperial units.
- Estimate the volume or capacity of a 3-D object or container, using a referent.
- Identify a situation where a given SI or imperial volume unit would be used.
- Convert a volume measurement expressed in one SI unit cubed to another SI unit cubed.
- Convert a volume measurement expressed in one imperial unit cubed to another imperial unit cubed.
- Describe the relationship between the volumes of
 - cones and cylinders with the same base and height
 - pyramids and prisms with the same base and height
- Determine the volume of cones, cylinders, prisms, pyramids, spheres, and composite 3-D objects, using a variety of measuring tools such as rulers, tape measures, calipers, or micrometers.
- Determine the capacity of cones, cylinders, prisms, pyramids, and spheres using a variety of measuring tools such as graduated cylinders, measuring cups, or measuring spoons.
- Illustrate, using examples, the effect of dimensional changes on volume.
- Solve a contextual problem that involves the volume of a 3-D object, including composite 3-D objects or the capacity of a container.
- Solve a contextual problem that involves the volume of a 3-D object and requires the manipulation of formulas.

Grade 11 Essential Mathematics – Half Course III

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:

3-D Geometry (*continued*)

General Learning Outcome:

Develop an understanding of spatial relationships applied to volume and surface area.

Specific Learning Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 11E3.G.3. Solve problems that require the manipulation and application of formulas related to
- volume and capacity
 - surface area
- [CN, PS, R]

- Solve a contextual problem involving the application of a formula that does not require manipulation.
- Solve a contextual problem involving the application of a formula that requires manipulation.
- Explain or verify that different forms of the same formula are equivalent.
- Describe, using examples, how a given formula is used in a trade or an occupation.
- Create and solve a contextual problem that involves a formula.
- Identify and correct errors in a solution to a problem that involves a formula.

Grade 11 Essential Mathematics – Half Course III

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Statistics

General Learning Outcome:
Develop statistical reasoning.

Specific Learning Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11E3.S.1.	Solve problems that involve creating and interpreting graphs, including ■ bar graphs ■ histograms ■ line graphs ■ circle graphs [C, CN, PS, R, T, V]	■ Determine the possible graphs that can be used to represent a data set, and explain the advantages and disadvantages of each. ■ Create, with or without technology, a graph to represent a data set. ■ Describe the trends in the graph of a data set. ■ Interpolate or extrapolate values from a graph. ■ Explain, using examples, how the same graph can be used to justify more than one conclusion. ■ Explain, using examples, how different graphic representations of the same data set can be used to emphasize a point of view. ■ Solve a contextual problem that involves the interpretation of a graph.
-----------	---	---

Grade 11 Essential Mathematics – Half Course IV

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Analysis of Games and Numbers

General Learning Outcome:
Develop critical-thinking skills.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 11E4.A.1. Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.
[C, CN, PS, R]

It is intended that this learning outcome be integrated throughout the course by using puzzles and games such as cribbage, magic squares, and Kakuro.

- Determine, explain, and verify a strategy to solve a puzzle or to win a game such as
 - guess and check
 - look for a pattern
 - make a systematic list
 - draw or model
 - eliminate possibilities
 - simplify the original problem
 - work backward
 - develop alternative approaches
- Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.
- Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Grade 11 Essential Mathematics – Half Course IV

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Managing Money		General Learning Outcome: Develop an understanding of managing money.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
11E4.M.1.	Solve problems that involve personal budgets. [CN, PS, R, T]	■ Identify income and expenses that should be included in a personal budget. ■ Explain considerations that must be made when developing a budget, including prioritizing, recurring, and unexpected expenses. ■ Create a personal budget, with or without technology, based on given income and expense data. ■ Collect income and expense data, and create a budget. ■ Modify a budget to achieve a set of personal goals. ■ Investigate and analyze, with or without technology, “what if ...” questions related to personal budgets.
11E4.M.2.	Demonstrate an understanding of financial institution services used to access and manage finances. [C, CN, R, T]	■ Describe the type of banking services available from various financial institutions, such as online services. ■ Describe the types of accounts and related service charges available at various financial institutions. ■ Identify the type of account that best meets the needs for a given set of criteria. ■ Identify and explain, for different accounts, the various record-keeping options such as deposit slips, withdrawal slips, cancelled cheques, account statements, cheque register, and receipts. ■ Identify and explain various automated teller machine (ATM) service charges. ■ Describe the advantages and disadvantages of online banking. ■ Describe the advantages and disadvantages of debit card purchases. ■ Describe ways that ensure the security of personal and financial information, such as passwords, encryption, protection of personal identification number (PIN), and other personal identity information.

Grade 11 Essential Mathematics – Half Course IV

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Relations and Patterns		General Learning Outcome: Develop proportional reasoning.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
11E4.R.1.	Demonstrate an understanding of slope ■ as rise over run ■ as rate of change by solving problems. [C, CN, PS, V]	■ Describe contexts that involve slope, such as ramps, roofs, road grade, flow rates within a tube, skateboard parks, or ski hills. ■ Explain, using diagrams, the difference between two slopes, and describe the implications. ■ Describe the conditions under which a slope will be either zero or undefined. ■ Explain, using examples and illustrations, slope as rise over run. ■ Verify that the slope of an object, such as a ramp or a roof, is constant. ■ Explain, using illustrations, the relationship between slope and angle of elevation. ■ Explain the implications, such as safety and functionality, of different slopes in a context. ■ Explain, using examples and illustrations, slope as rate of change. ■ Solve a contextual problem that involves slope.
11E4.R.2.	Solve problems by applying proportional reasoning and unit analysis. [C, CN, PS, R]	■ Explain the process of unit analysis used to solve a problem involving proportions such as km/h, or revolutions per minute. ■ Solve a problem, using unit analysis. ■ Explain, using an example, how unit analysis and proportional reasoning are related. ■ Solve a problem within or between systems, using proportions or tables.

Grade 11 Essential Mathematics – Half Course IV

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Relations and Patterns <i>(continued)</i>		General Learning Outcome: Develop proportional reasoning.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
11E4.R.3.	Solve problems that require the manipulation and application of formulas related to slope and rate of change. [CN, PS, R]	■ Solve a contextual problem involving the application of a formula that does not require manipulation. ■ Solve a contextual problem involving the application of a formula that requires manipulation. ■ Explain or verify that different forms of the same formula are equivalent. ■ Describe, using examples, how a given formula is used in a trade or an occupation. ■ Create and solve a contextual problem that involves a formula. ■ Identify and correct errors in a solution to a problem that involves a formula.
11E4.R.4.	Solve problems that involve scale. [PS, R, T, V]	■ Describe contexts in which a scale representation is used. ■ Determine, using proportional reasoning, the dimensions of an object from a given scale drawing or model. ■ Construct a model of a 3-D object, given the scale. ■ Draw, with or without technology, a scale diagram of an object. ■ Solve a contextual problem that involves scale.

Grade 11 Essential Mathematics – Half Course IV

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Relations and Patterns (<i>continued</i>)		General Learning Outcome: Develop proportional reasoning.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
11E4.R.5.	Demonstrate an understanding of linear relations by ■ recognizing patterns and trends ■ graphing ■ creating tables of values ■ writing equations ■ interpolating and extrapolating ■ solving problems [CN, PS, R, T, V]	■ Identify and describe the characteristics of a linear relation represented in a graph, table of values, number pattern, or equation. ■ Sort a set of graphs, tables of values, number patterns, or equations into linear and non-linear relations. ■ Write an equation for a context involving direct or partial variation. ■ Create a table of values for an equation of a linear relation. ■ Sketch the graph for a table of values. ■ Explain why the points should or should not be connected on the graph for a context. ■ Create, with or without technology, a graph to represent a data set. ■ Describe the trends in the graph of a data set. ■ Sort a set of scatterplots according to the trends represented (linear, nonlinear, or no trend). ■ Solve a contextual problem that requires interpolation or extrapolation. ■ Relate slope and rate of change to linear relations. ■ Match contexts with their corresponding graphs, and explain the reasoning. ■ Solve a contextual problem involving the application of a formula for a linear relation.

Grade 11 Essential Mathematics – Half Course IV

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Trigonometry

General Learning Outcome:
Develop spatial sense related to triangles.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11E4.TG.1. Solve problems that involve two and three right triangles.
[CN, PS, T, V]

- Identify all of the right triangles in a given illustration for a context.
- Determine whether a solution to a problem that involves two or three right triangles is reasonable.
- Sketch a representation of a descriptive problem in a 2-D or 3-D context.
- Solve a contextual problem that involves angles of elevation or angles of depression.
- Solve a contextual problem that involves two or three right triangles, using the primary trigonometric ratios.

Grade 11 Essential Mathematics – Half Course IV

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Design Modelling		General Learning Outcome: Develop spatial sense.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
11E4.D.1.	Model and draw 3-D objects and their views. [CN, R, V]	■ Draw a 2-D representation of a 3-D object. ■ Draw, using isometric dot paper, a 3-D object. ■ Draw to scale top, front, and side views of a 3-D object. ■ Construct a model of a 3-D object, given the top, front, and side views. ■ Draw a 3-D object, given the top, front, and side views. ■ Determine if views of a 3-D object represent the object, and explain the reasoning. ■ Identify the point of perspective of a one-point perspective drawing of a 3-D object. ■ Draw a one-point perspective view of a 3-D object.
11E4.D.2.	Draw and describe exploded views, component parts, and scale diagrams of simple 3-D objects. [CN, V]	■ Draw the component parts of an exploded diagram, and explain their relationship to the original 3-D object. ■ Sketch an exploded view of a 3-D object to represent the component parts. ■ Draw to scale the component parts of a 3-D object. ■ Sketch a 2-D representation of a 3-D object, given its exploded view. ■ Solve a contextual problem that involves scale.

GRADE 11 PRE-CALCULUS MATHEMATICS (30S)

Grade 11 Pre-Calculus Mathematics (30S) is designed for students who intend to study calculus and related mathematics as part of post-secondary education. It builds on the topics studied in Grade 10 Introduction to Applied and Pre-Calculus Mathematics and provides background knowledge and skills for Grade 12 Pre-Calculus Mathematics.

The course comprises a high-level study of theoretical mathematics with an emphasis on problem solving and mental mathematics. The topics include study of algebra, quadratic functions, reciprocal functions, and trigonometry.

Assessment of Grade 11 Pre-Calculus Mathematics should be a balance of assessment *for* learning, assessment *as* learning, and assessment *of* learning. Assessment tools used in Grade 11 Pre-Calculus Mathematics should be varied and may include observation, homework, learning conversations or interviews, summative unit essays, demonstrations, presentations, performance tasks, learning logs, projects, investigations, reflective journals, portfolios, quizzes, tests, and examinations. An

appropriately prepared portfolio requires a consistent effort throughout the school term and a commitment to completing quality work on a daily basis.

The learning outcomes are divided into three topics: Algebra and Number; Trigonometry; and Relations and Functions. For instructional purposes, the learning outcomes could be arranged into units. Learning outcomes from different topics could be taught in the same unit. Some learning outcomes may fit into multiple units, and parts of the learning outcome could be taught in one unit while the remaining parts can be taught later. Two possible sequences of the learning outcomes into units with suggested time allotments follow. The suggested times include time for instruction and assessment. These are not the only possibilities, but they will provide some direction for teachers for their first time through the course.

Regardless of the organization of the learning outcomes into units, students should constantly be looking for and be given opportunities to see connections between the various learning outcomes in Grade 11 Pre-Calculus Mathematics.

Possibility 1			Possibility 2		
Unit	Learning Outcomes	Suggested Hours	Unit	Learning Outcomes	Suggested Hours
Quadratic Equations	R1, R5	12	Algebra	R1, A4, A5	15
Radicals	A2, A3	15	Functions	R3, R4, R11	20
Quadratic Functions	R3, R4	17	Sequences	R9, R10	10
Sequences	R9, R10	10	Absolute Value	A1, R2	10
Rationals	A4, A5, A6, R11	15	Equation Solving	A3, A6, R5, R6	15
Trigonometry	T1, T2, T3	20	Trigonometry	T1, T2, T3	20
Systems	R6	6	Inequalities	R7, R8	10
Inequalities	A1, R2, R7, R8	15	Radicals	A2, A3	10
Total		110	Total		110

General and Specific Learning Outcomes with Achievement Indicators by Course

Grade 11 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Algebra and Number		General Learning Outcome: Develop algebraic reasoning and number sense.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
11P.A.1.	Demonstrate an understanding of the absolute value of real numbers. [ME, R, V]	■ Determine the distance of two real numbers of the form $\pm a$, $a \in R$, from 0 on a number line, and relate this to the absolute value of a (a). ■ Determine the absolute value of a positive or negative real number. ■ Explain, using examples, how distance between two points on a number line can be expressed in terms of absolute value. ■ Determine the absolute value of a numerical expression. ■ Compare and order the absolute values of real numbers in a set.
11P.A.2.	Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands. [CN, ME, PS, R, T]	■ Compare and order a set of radical expressions with numerical radicands. ■ Express an entire radical with a numerical radicand as a mixed radical. ■ Express a mixed radical with a numerical radicand as an entire radical. ■ Perform one or more operations to simplify radical expressions with numerical or variable radicands. ■ Rationalize the denominator of a rational expression with monomial or binomial denominators. ■ Describe the relationship between rationalizing a binomial denominator of a rational expression and the product of the factors of a difference of squares expression. ■ Explain, using examples, that $(-x)^2 = x^2$, $\sqrt{x^2} = x$, if $x^2 = a$, then $x = \pm\sqrt{a}$ ■ Identify the values of the variable for which a radical expression is defined. ■ Solve a problem that involves radical expressions.

Grade 11 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Algebra and Number <i>(continued)</i>		General Learning Outcome: Develop algebraic reasoning and number sense.
Specific Learning Outcomes <i>It is expected that students will:</i>	Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>	
11P.A.3. Solve problems that involve radical equations (limited to square roots). [C, CN, PS, R, T]	■ Determine any restrictions on values for the variable in a radical equation. ■ Determine the roots of a radical equation algebraically, and explain the process used to solve the equation. ■ Verify, by substitution, that the values determined in solving a radical equation algebraically are roots of the equation. ■ Demonstrate that some roots determined in solving a radical equation algebraically are extraneous. ■ Solve a problem by modelling a situation using a radical equation.	
11P.A.4. Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials). [C, ME, R]	■ Compare the strategies for writing equivalent forms of rational expressions to the strategies for writing equivalent forms of rational numbers. ■ Explain why a value is non-permissible for a rational expression. ■ Determine the non-permissible values for a rational expression. ■ Determine a rational expression that is equivalent to a rational expression by multiplying the numerator and denominator by the same factor (limited to a monomial or a binomial), and state the non-permissible values of the equivalent rational expression. ■ Simplify a rational expression. ■ Explain why the non-permissible values of a rational expression and its simplified form are the same. ■ Identify and correct errors in a simplification of a rational expression, and explain the reasoning.	

Grade 11 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Algebra and Number (<i>continued</i>)		General Learning Outcome: Develop algebraic reasoning and number sense.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
11P.A.5.	Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials). [C, CN, ME, R]	■ Compare the strategies for performing an operation on rational expressions to the strategies for performing the same operation on rational numbers. ■ Determine the non-permissible values when performing operations on rational expressions. ■ Determine, in simplified form, the sum or difference of rational expressions with the same denominator. ■ Determine, in simplified form, the sum or difference of rational expressions in which the denominators are not the same and which may or may not contain common factors. ■ Determine, in simplified form, the product or quotient of rational expressions. ■ Simplify an expression that involves two or more operations on rational expressions.
11P.A.6.	Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials, or trinomials). [C, CN, PS, R]	■ Determine the non-permissible values for the variable in a rational equation. ■ Determine the solution to a rational equation algebraically, and explain the process. ■ Explain why a value obtained in solving a rational equation may not be a solution of the equation. ■ Solve a problem by modelling a situation using a rational equation.
	It is intended that the rational equations be those that can be simplified to linear and quadratic equations.	

Grade 11 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Trigonometry

General Learning Outcome:
Develop trigonometric reasoning.

Specific Learning Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11P.T.1.	Demonstrate an understanding of angles in standard position [0° to 360°]. [C, R, V]	■ Sketch an angle in standard position, given the measure of the angle. ■ Determine the reference angle for an angle in standard position. ■ Explain, using examples, how to determine the angles from 0° to 360° that have the same reference angle as a given angle. ■ Illustrate, using examples, that any angle from 90° to 360° is the reflection in the x-axis and/or the y-axis of its reference angle. ■ Determine the quadrant in which an angle in standard position terminates. ■ Draw an angle in standard position given any point P (x, y) on the terminal arm of the angle. ■ Illustrate, using examples, that the points P (x, y), P (-x, y), P (-x, -y), and P (x, -y) are points on the terminal sides of angles in standard position that have the same reference angle.
----------	--	---

Grade 11 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Trigonometry (*continued*)

General Learning Outcome:
Develop trigonometric reasoning.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators
*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11PT.2.	Solve problems, using the three primary trigonometric ratios (sine, cosine, and tangent) for angles from 0° to 360° in standard position. [C, ME, PS, R, T, V]	■ Determine, using the Pythagorean theorem or the distance formula, the distance from the origin to a point $P(x, y)$ on the terminal arm of an angle. ■ Determine the value of $\sin \theta$, $\cos \theta$, or $\tan \theta$, given any point $P(x, y)$ on the terminal arm of angle θ. ■ Determine, without the use of technology, the value of $\sin \theta$, $\cos \theta$, or $\tan \theta$, given any point $P(x, y)$ on the terminal arm of angle θ, where $\theta = 0^\circ, 90^\circ, 180^\circ, 270^\circ$, or 360°. ■ Determine the sign of a trigonometric ratio for an angle, without the use of technology, and explain. ■ Solve an equation of the form $\sin \theta = a$ or $\cos \theta = a$, where $-1 \leq a \leq 1$, or an equation of the form $\tan \theta = a$, where a is a real number. ■ Determine the exact value of the sine, cosine, or tangent of an angle with a reference angle of $30^\circ, 45^\circ$, or 60°. ■ Describe patterns in and among the values of the sine, cosine, and tangent ratios for angles from 0° to 360°. ■ Sketch a diagram to represent a problem involving trigonometric ratios. ■ Solve a contextual problem, using trigonometric ratios.
11PT.3.	Solve problems, using the cosine law and sine law, including the ambiguous case. [C, CN, PS, R, T]	■ Sketch a diagram to represent a problem that involves a triangle without a right angle. ■ Solve a non-right triangle using right triangle methods. ■ Explain the steps in a given proof of the sine law or cosine law. ■ Sketch a diagram and solve a contextual problem, using the cosine law. ■ Sketch a diagram and solve a contextual problem, using the sine law. ■ Describe and explain ambiguous case problems that may have no solution, one solution, or two solutions.

Grade 11 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions

General Learning Outcome:
Develop algebraic and graphical reasoning through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11P.R.1. Factor polynomial expressions of the form

- $ax^2 + bx + c, a \neq 0$
- $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$
- $a(f(x))^2 + b(f(x)) + c, a \neq 0$
- $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$

where a, b , and c are rational numbers.
[ME, R]

- Factor a polynomial expression that requires the identification of common factors.
- Determine whether a binomial is a factor for a polynomial expression, and explain why or why not.
- Factor a polynomial expression of the form
 - $ax^2 + bx + c, a \neq 0$
 - $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$
- Factor a polynomial expression that has a quadratic pattern, including
 - $a(f(x))^2 + b(f(x)) + c, a \neq 0$
 - $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$

Grade 11 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning
through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11P.R.2. Graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems.
[C, PS, R, T, V]

- Create a table of values for $y = |f(x)|$, given a table of values for $y = f(x)$.
- Generalize a rule for writing absolute value functions in piecewise notation.
- Sketch the graph of $y = |f(x)|$; state the intercepts, domain and range; and explain the strategy used.
- Solve absolute value equations graphically, with or without technology.
- Solve, algebraically, equations with a single absolute value, and verify the solution.
- Explain why the absolute value equation $|f(x)| = a$, $a < 0$ has no solution.
- Determine and correct errors in a solution to an absolute value equation.
- Solve a problem that involves absolute value functions.

Grade 11 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11P.R.3.	Analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the ■ vertex ■ domain and range ■ direction of opening ■ axis of symmetry ■ x- and y-intercepts [C, CN, R, T, V]	■ Explain why a function given in the form $y = a(x - p)^2 + q$ is a quadratic function. ■ Compare the graphs of a set of functions of the form $y = ax^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of a. ■ Compare the graphs of a set of functions of the form $y = x^2 + q$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of q. ■ Compare the graphs of a set of functions of the form $y = (x - p)^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of p. ■ Determine the coordinates of the vertex for a quadratic function of the form $y = a(x - p)^2$, and verify with or without technology. ■ Generalize, using inductive reasoning, a rule for determining the coordinates of the vertex for quadratic functions of the form $y = a(x - p)^2 + q$. ■ Sketch the graph of $y = a(x - p)^2 + q$, using transformations, and identify the vertex, domain and range, direction of opening, axis of symmetry, and x- and y-intercepts. ■ Explain, using examples, how the values of a and q may be used to determine whether a quadratic function has zero, one, or two x-intercepts. ■ Write a quadratic function in the form $y = a(x - p)^2 + q$ for a graph or a set of characteristics of a graph.
----------	---	--

Grade 11 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning
through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11P.R.4.	Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including ■ vertex ■ domain and range ■ direction of opening ■ axis of symmetry ■ x- and y-intercepts [C, CN, PS, R, T, V]	■ Explain the reasoning for the process of completing the square, as shown in an example. ■ Write a quadratic function given in the form $y = ax^2 + bx + c$ as a quadratic function in the form $y = a(x - p)^2 - q$ by completing the square. ■ Identify, explain, and correct errors in an example of completing the square. ■ Determine the characteristics of a quadratic function given in the form $y = ax^2 + bx + c$, and explain the strategy used. ■ Sketch the graph of a quadratic function given in the form $y = ax^2 + bx + c$. - n Verify, with or without technology, that a quadratic function in the form $y = ax^2 + bx + c$ represents the same function as the quadratic function in the form $y = a(x - p)^2 + q$. ■ Write a quadratic function that models a situation, and explain any assumptions made. ■ Solve a problem, with or without technology, by analyzing a quadratic function.
----------	--	---

Grade 11 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning
through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11P.R.5. Solve problems that involve quadratic equations.
[C, CN, PS, R, T, V]

- Explain, using examples, the relationship among the roots of a quadratic equation, the zeros of the corresponding quadratic function, and the x-intercepts of the graph of the quadratic function.
- Derive the quadratic formula, using deductive reasoning.
- Solve a quadratic equation of the form $ax^2 + bx + c = 0$ by using strategies such as
 - determining square roots
 - factoring
 - completing the square
 - applying the quadratic formula
 - graphing its corresponding function
- Select a method for solving a quadratic equation, justify the choice, and verify the solution.
- Explain, using examples, how the discriminant may be used to determine whether a quadratic equation has two, one, or no real roots; and relate the number of zeros to the graph of the corresponding quadratic function.
- Identify and correct errors in a solution to a quadratic equation.
- Solve a problem by determining or analyzing a quadratic equation.

Grade 11 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions *(continued)*

General Learning Outcome:
Develop algebraic and graphical reasoning
through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 11P.R.6. Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables.
[C, CN, PS, R, T, V]

It is intended that the quadratic equations be limited to those that correspond to quadratic functions.

- Model a situation, using a system of linear-quadratic or quadratic-quadratic equations.
- Relate a system of linear-quadratic or quadratic-quadratic equations to the context of a problem.
- Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations graphically, with technology.
- Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations algebraically.
- Explain the meaning of the points of intersection of a system of linear-quadratic or quadratic-quadratic equations.
- Explain, using examples, why a system of linear-quadratic or quadratic-quadratic equations may have zero, one, two, or an infinite number of solutions.
- Solve a problem that involves a system of linear-quadratic or quadratic-quadratic equations, and explain the strategy used.

Grade 11 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Relations and Functions (<i>continued</i>)		General Learning Outcome: Develop algebraic and graphical reasoning through the study of relations.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
11P.R.7.	Solve problems that involve linear and quadratic inequalities in two variables. [C, PS, T, V]	■ Explain, using examples, how test points can be used to determine the solution region that satisfies an inequality. ■ Explain, using examples, when a solid or broken line should be used in the solution for an inequality. ■ Sketch, with or without technology, the graph of a linear or quadratic inequality. ■ Solve a problem that involves a linear or quadratic inequality.
11P.R.8.	Solve problems that involve quadratic inequalities in one variable. [CN, PS, V]	■ Determine the solution of a quadratic inequality in one variable, using strategies such as case analysis, graphing the related function, roots and test points, or sign analysis; and explain the strategy used. ■ Represent and solve a problem that involves a quadratic inequality in one variable. ■ Interpret the solution to a problem that involves a quadratic inequality in one variable.
11P.R.9.	Analyze arithmetic sequences and series to solve problems. [C, CN, PS, R, T]	■ Identify the assumption(s) made when defining an arithmetic sequence or series. ■ Provide and justify an example of an arithmetic sequence. ■ Derive a rule for determining the general term of an arithmetic sequence. ■ Describe the relationship between arithmetic sequences and linear functions. ■ Determine the first term, the common difference, the number of terms, or the value of a specific term in a problem involving an arithmetic sequence. ■ Derive a rule for determining the sum of n terms of an arithmetic series. ■ Determine the first term, the common difference, the number of terms, or the value of the sum of specific numbers of terms in a problem involving an arithmetic series. ■ Solve a problem that involves an arithmetic sequence or series.

Grade 11 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning
through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

11P.R.10. Analyze geometric sequences and series to solve problems.
[C, CN, PS, R, T]

- Identify assumptions made when identifying a geometric sequence or series.
- Provide and justify an example of a geometric sequence.
- Derive a rule for determining the general term of a geometric sequence.
- Determine the first term, the common ratio, the number of terms, or the value of a specific term in a problem involving a geometric sequence.
- Derive a rule for determining the sum of n terms of a geometric series.
- Determine the first term, the common ratio, the number of terms, or the value of the sum of a specific number of terms in a problem involving a geometric series.
- Generalize, using inductive reasoning, a rule for determining the sum of an infinite geometric series.
- Explain why an infinite geometric series is convergent or divergent.
- Solve a problem that involves a geometric sequence or series.

11P.R.11. Graph and analyze reciprocal functions (limited to the reciprocal of linear and quadratic functions).
[CN, R, T, V]

- Compare the graph of $y = \frac{1}{f(x)}$ to the graph of $y = f(x)$.
- Identify, given a function $f(x)$, values of x for which $y = \frac{1}{f(x)}$ will have vertical asymptotes; and describe their relationship to the non-permissible values of the related rational expression.
- Graph, with or without technology, $y = \frac{1}{f(x)}$, given $y = f(x)$ as a function or a graph, and explain the strategies used.
- Graph, with or without technology, $y = f(x)$, given $y = \frac{1}{f(x)}$ as a function or a graph, and explain the strategies used.

GRADE 12 APPLIED MATHEMATICS (40S)

Grade 12 Applied Mathematics (40S) is intended for students considering post-secondary studies that do not require a study of theoretical calculus. It is context-driven and promotes the learning of numerical and geometrical problem-solving techniques as they relate to the world around us.

The primary goals of Applied Mathematics are to have students develop critical-thinking skills through problem solving and through modelling real-world situations mathematically to make predictions.

These goals may be attained in a number of ways. Students may collect data in experiments and activities to develop mathematical concepts by analyzing that data. They are encouraged to learn and demonstrate effective communication skills through a variety of media. Students are expected to become proficient in both oral and written communication skills.

Applied Mathematics is designed to promote student flexibility and responsibility. Flexibility is encouraged by having students work on non-routine problems and projects. Responsibility is encouraged as students work individually and in cooperative groups to explore connections with other mathematical areas, school subjects, and real-life applications.

Technology is an integral part of both learning and assessment in Applied Mathematics. Graphing calculators, spreadsheets, or other computer software will be used by students for mathematical explorations, modelling, and problem solving.

Assessment of Grade 12 Applied Mathematics should be a balance of assessment *for* learning, assessment *as* learning, and assessment *of* learning. Assessment tools used in Grade 12 Applied Mathematics should be varied and may include observation, homework, learning conversations or interviews, summative unit essays, demonstrations, presentations, performance tasks, learning logs, projects, investigations, reflective journals, portfolios, quizzes, tests, and examinations. An appropriately prepared portfolio requires a consistent effort throughout the school term and a commitment to completing quality work on a daily basis.

The Grade 12 Applied Mathematics course includes the following topics: Financial Mathematics, Logical Reasoning, Probability, Relations and Functions, and Design and Measurement. Additionally, students will complete a Mathematics Research Project.

Learning outcomes from various topic areas may be combined in any order when considering learning activities. The suggested time includes instructional and assessment time. Two possible teaching sequences and suggested hours are presented in the following tables:

Possibility 1			Possibility 2		
Unit	Learning Outcomes	Suggested Hours	Unit	Learning Outcomes	Suggested Hours
Sinusoidal Functions	L1, R3	12	Analysis of Games and Numbers	L1	5 (integrated throughout course)
Compound Interest	L1, FM1, FM2	12	Probability	P1, P2, P3	15
Probability	L1, L2, L3, P1, P2, P3	20	Personal Finance	FM1, FM2, FM3	23
Polynomial Functions	L1, R1	12	Functions	R1, R2	20
Permutations and Combinations	L1, P4, P5, P6	12	Logic	L2, L3	7
Investments	L1, FM3	12	Permutations and Combinations	P4, P5, P6	10
Design and Measurement	L1, D1	10	Sinusoidal Functions	R3	10
Research Project	L1, RP1	10	Design and Measurement	D1	10
Exponential and Logarithmic Functions	L1, R2	10	Research Project	RP1	10
Total		110	Total		110

General and Specific Learning Outcomes with Achievement Indicators by Course

Grade 12 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Financial Mathematics		General Learning Outcome: Develop number sense in financial applications.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
12A.FM.1. Solve problems that involve compound interest in financial decision making. [C, CN, PS, T, V]		■ Explain the advantages and disadvantages of compound interest and simple interest. ■ Identify situations involving compound interest. ■ Graph and compare the total interest paid or earned for different compounding periods. ■ Determine, given the principal, interest rate, and number of compounding periods, the total interest of a loan. ■ Graph and describe the effects of changing the value of one of the variables in a situation involving compound interest. ■ Determine, using technology, the total cost of a loan under a variety of conditions, such as different amortization periods, interest rates, and compounding periods or terms. ■ Compare and explain, using technology, different credit options involving compound interest, including bank or store credit cards or special promotions. ■ Solve a contextual problem involving compound interest.

Grade 12 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Financial Mathematics (<i>continued</i>)		General Learning Outcome: Develop number sense in financial applications.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
12A.FM.2.	Analyze costs and benefits of renting, leasing, and buying. [CN, PS, R, T]	- Identify and describe examples of assets that appreciate or depreciate. - Compare, using examples, renting, leasing, and buying. - Justify, for a specific set of circumstances, whether renting, buying, or leasing would be advantageous. - Solve, using technology, a contextual problem involving renting, leasing, or buying. - Solve, using technology, a contextual problem involving cost-and-benefit analysis.
12A.FM.3.	Analyze an investment portfolio in terms of - interest rate - rate of return - total return [ME, PS, R, T] Students should be able to solve an investment problem using tools such as debt-to-equity ratio, gross debt service ratio, or net worth.	- Determine, using technology, the total value of an investment when there are regular contributions to the principal. - Graph and compare the total value of an investment with or without regular contributions. - Apply the Rule of 72 to solve investment problems, and explain the limitations of the rule. - Determine, using technology, possible investment strategies to achieve a financial goal. - Explain the advantages and disadvantages of long-term or short-term investment options. - Explain, using examples, why smaller investments over a longer term may be better than larger investments over a shorter term. - Determine and compare the strengths and weaknesses of two or more portfolios. - Solve an investment problem.

Grade 12 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Logical Reasoning

General Learning Outcome:
Develop logical reasoning.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 12A.L.1. Analyze puzzles and games that involve numerical and logical reasoning, using problem-solving strategies.
[CN, ME, PS, R, T]

It is intended that this learning outcome be integrated throughout the course by using games and puzzles such as chess, Sudoku, Nim, logic puzzles, magic squares, Kakuro, and cribbage.

- Determine, explain, and verify a strategy to solve a puzzle or to win a game such as
 - guess and check
 - look for a pattern
 - make a systematic list
 - draw or model
 - eliminate possibilities
 - simplify the original problem
 - work backward
 - develop alternative approaches
- Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.
- Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Grade 12 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Logical Reasoning (<i>continued</i>)		General Learning Outcome: Develop logical reasoning.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
12A.L.2.	Solve problems that involve the application of set theory. [CN, PS, R, T, V]	■ Explain how set theory is used in applications such as Internet searches, database queries, data analysis, games, and puzzles. ■ Provide examples of the empty set, disjoint sets, subsets, and universal sets in context, and explain the reasoning. ■ Organize information such as collected data and number properties, using graphic organizers, and explain the reasoning. ■ Explain what a specified region in a Venn diagram represents. ■ Determine the elements in the complement, the intersection, or the union of two sets. ■ Identify and correct errors in a solution to a problem that involves sets. ■ Solve a contextual problem that involves sets, and record the solution.
12A.L.3.	Solve problems that involve conditional statements. [C, CN, PS, R, T]	■ Analyze an “if-then” statement, make a conclusion, and explain the reasoning. ■ Make and justify a decision, using “what if?” questions, in contexts such as probability, finance, sports, games, or puzzles, with or without technology. ■ Determine the converse, inverse, and contrapositive of an “if-then” statement, determine its truth, and, if it is false, provide a counter-example. ■ Demonstrate, using examples, that the truth of any statement does not imply the truth of its converse or inverse. ■ Demonstrate, using examples, that the truth of any statement does imply the truth of its contrapositive. ■ Identify and describe contexts in which a biconditional statement can be justified. ■ Analyze and summarize, using a graphic organizer such as a truth table or Venn diagram, the possible results of given logical arguments that involve biconditional, converse, inverse, or contrapositive statements.

Grade 12 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Probability

General Learning Outcome:

Develop critical thinking skills related to uncertainty.

Specific Learning Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12A.P.1.	Interpret and assess the validity of odds and probability statements. [C, CN, ME, T]	■ Provide examples of statements of probability and odds found in fields such as media, biology, sports, medicine, weather, sociology, or psychology. ■ Explain, using examples, the relationship between odds (part-part) and probability (part-whole). ■ Express odds as a probability and vice versa. ■ Determine the probability of, or the odds for and against, an outcome in a situation. ■ Explain, using examples, how decisions may be based on probability or odds and on subjective judgments. ■ Solve a contextual problem that involves odds or probability.
12A.P.2.	Solve problems that involve the probability of mutually exclusive and non-mutually exclusive events. [CN, PS, R, T, V]	■ Classify events as mutually exclusive or non-mutually exclusive, and explain the reasoning. ■ Determine whether two events are complementary, and explain the reasoning. ■ Represent, using set notation or graphic organizers, mutually exclusive (including complementary) and non-mutually exclusive events. ■ Solve a contextual problem that involves the probability of mutually exclusive or non-mutually exclusive events. ■ Solve a contextual problem that involves the probability of complementary events. ■ Create and solve a problem that involves mutually exclusive or non-mutually exclusive events.

Grade 12 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Probability (<i>continued</i>)		General Learning Outcome: Develop critical thinking skills related to uncertainty.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
12A.P.3.	Solve problems that involve the probability of independent and dependent events. [CN, PS, R, T]	■ Compare, using examples, dependent and independent events. ■ Determine the probability of an event, given the occurrence of a previous event. ■ Determine the probability of two dependent or two independent events. ■ Create and solve a contextual problem that involves determining the probability of dependent or independent events.
12A.P.4.	Solve problems that involve the fundamental counting principle. [PS, R, T, V]	■ Represent and solve counting problems, using a graphic organizer. ■ Generalize, from examples, the fundamental counting principle. ■ Identify and explain assumptions made in solving a counting problem. ■ Solve a contextual counting problem, using the fundamental counting principle, and explain the reasoning.
12A.P.5.	Solve problems that involve permutations. [ME, PS, R, T, V] Students should be able to use strategies such as cases or grouping objects together to solve a contextual problem. Also, it is intended that circular permutations not be included.	■ Represent the number of arrangements of n elements taken n at a time, using factorial notation. ■ Determine the value of a factorial. ■ Simplify a numeric fraction containing factorials in both the numerator and denominator. ■ Determine the number of permutations of n elements taken r at a time. ■ Determine the number of permutations of n elements taken n at a time where some elements are not distinct. ■ Explain, using examples, the effect on the total number of permutations of n elements when two or more elements are identical. ■ Generalize strategies for determining the number of permutations of n elements taken r at a time. ■ Solve a contextual problem that involves probability and permutations.

Grade 12 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:

Probability (*continued*)

General Learning Outcome:

Develop critical thinking skills related to uncertainty.

Specific Learning Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12A.P.6. Solve problems that involve combinations.
[ME, PS, R, T, V]

- Explain, using examples, why order is or is not important when solving problems that involve permutations or combinations.
- Determine the number of combinations of n elements taken r at a time.
- Generalize strategies for determining the number of combinations of n elements taken r at a time.
- Solve a contextual problem that involves probability and combinations.

Grade 12 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions

General Learning Outcome:
Develop algebraic and graphical reasoning through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12A.R.1.	Represent data, using polynomial functions (of degree ≤ 3), to solve problems. [C, CN, PS, T, V]	■ Describe, orally and in written form, the characteristics of polynomial functions by analyzing their graphs or their equations. ■ Match equations in a set to their corresponding graphs. ■ Graph data and determine the polynomial function that best approximates the data. ■ Interpret the graph of a polynomial function that models a situation, and explain the reasoning. ■ Solve, using technology, a contextual problem that involves data that is best represented by graphs of polynomial functions, and explain the reasoning.
12A.R.2.	Represent data, using exponential and logarithmic functions, to solve problems. [C, CN, PS, T, V]	■ Describe, orally and in written form, the characteristics of exponential or logarithmic functions by analyzing their graphs or their equations. ■ Match equations in a set to their corresponding graphs. ■ Graph data and determine the exponential or logarithmic function that best approximates the data. ■ Interpret the graph of an exponential or logarithmic function that models a situation, and explain the reasoning. ■ Solve, using technology, a contextual problem that involves data that is best represented by graphs of exponential or logarithmic functions, and explain the reasoning.

Grade 12 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning
through the study or relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether
students have met the corresponding specific learning outcome.*

12A.R.3.	Represent data, using sinusoidal functions, to solve problems. [C, CN, PS, T, V]	■ Describe, orally and in written form, the characteristics of sinusoidal functions by analyzing their graphs or their equations. ■ Match equations in a set to their corresponding graphs. ■ Graph data and determine the sinusoidal function that best approximates the data. ■ Interpret the graph of a sinusoidal function that models a situation, and explain the reasoning. ■ Solve, using technology, a contextual problem that involves data that is best represented by graphs of sinusoidal functions, and explain the reasoning.
----------	---	--

Grade 12 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Mathematics Research Project		General Learning Outcome: Develop an appreciation of the role of mathematics in society.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
12A.RP.1.	Research and give a presentation on a current event or an area of interest that involves mathematics. [C, CN, ME, PS, R, T, V]	■ Collect primary or secondary data (statistical or informational) related to the topic. ■ Assess the accuracy, reliability, and relevance of the primary or secondary data collected by ■ identifying examples of bias and points of view ■ identifying and describing the data collection methods ■ determining whether the data is relevant ■ determining whether the data is consistent with information obtained from other sources on the same topic ■ Interpret data, using statistical methods if applicable. ■ Identify controversial issues, if any, and present multiple sides of the issues with supporting data. ■ Organize and present the research project, with or without technology.

Grade 12 Applied Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Design and Measurement		General Learning Outcome: Develop critical thinking skills related to measurement design.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
12A.D. 1	Analyze objects, shapes, and processes to solve cost and design problems. [C, CN, ME, PS, R, T, V]	■ Solve a problem involving perimeter, area, and volume using dimensions and unit prices. ■ Solve a problem involving estimation and costing for objects, shapes, or processes when a design is given. ■ Identify and correct errors in a solution to a problem that involves costing for objects, shapes, or processes. ■ Estimate the solutions to complex measurement problems using simplified models. ■ Design an object, shape, layout, or process within a specified budget.

GRADE 12 ESSENTIAL MATHEMATICS (40S)

Grade 12 Essential Mathematics (40S) is intended for students whose post-secondary planning does not include a focus on mathematics and science-related fields. Grade 12 Essential Mathematics (40S) is a one-credit course consisting of two half-credits each emphasizing consumer applications, problem solving, decision making, and spatial sense.

Students are expected to work both individually and in small groups on mathematical concepts and skills encountered in everyday life in a technological society.

Assessment of Grade 12 Essential Mathematics should be a balance of assessment *for* learning, assessment *as* learning, and assessment *of* learning. Assessment tools used in

Grade 12 Essential Mathematics should be varied and may include observation, homework, learning conversations or interviews, summative unit essays, demonstrations, presentations, performance tasks, learning logs, projects, investigations, reflective journals, portfolios, quizzes, tests, and examinations. An appropriately prepared portfolio requires a consistent effort throughout the school term and a commitment to completing quality work on a daily basis.

The following tables list the units of study for each half-course along with an estimated number of hours for each unit. The time for each unit includes instructional and assessment time.

Grade 12 Essential Mathematics (45S) Half Course V		Grade 12 Essential Mathematics (45S) Half Course VI	
Unit	Suggested Hours	Unit	Suggested Hours
Analysis of Games and Numbers	7	Analysis of Games and Numbers	7
Vehicle Finance	17	Home Finance	12
Statistics	7	Geometry and Trigonometry	10
Precision Measurement	12	Business Finance	15
Career Life	12	Probability	11
Total	55	Total	55

General and Specific Learning Outcomes with Achievement Indicators by Course

Grade 12 Essential Mathematics – Half Course V

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Analysis of Games and Numbers		General Learning Outcome: Develop critical thinking skills.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
12E5.A.1.	Analyze puzzles and games that involve logical reasoning, using problem-solving strategies. [C, CN, PS, R]	■ Determine, explain, and verify a strategy to solve a puzzle or to win a game such as ■ guess and check ■ look for a pattern ■ make a systematic list ■ draw or model ■ eliminate possibilities ■ simplify the original problem ■ work backward ■ develop alternative approaches ■ Identify and correct errors in a solution to a puzzle or in a strategy for winning a game. ■ Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

It is intended that this learning outcome be integrated throughout the course by using puzzles and games such as Sudoku, Mastermind, Nim, and logic puzzles.

Grade 12 Essential Mathematics – Half Course V

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Vehicle Finance		General Learning Outcome: Develop an understanding of owning and operating a vehicle.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
12E5.V.1.	Solve problems that involve the acquisition, operation, and maintenance of a vehicle, when ■ buying ■ leasing ■ leasing to buy [C, CN, PS, R, T]	■ Describe and explain various options for buying, leasing, or leasing to buy a vehicle. ■ Determine costs associated with buying, leasing, or leasing to buy a new or used vehicle. ■ Solve, with or without technology, problems that involve the purchase, lease, or lease to purchase of a vehicle. ■ Determine costs associated with operating a vehicle including regular maintenance, repairs, fuel consumption, or depreciation. ■ Determine the costs of insuring a vehicle for various uses. ■ Justify a decision related to buying, leasing, or leasing to buy a vehicle, based on factors such as personal finances, intended use, maintenance, warranties, mileage, insurance, or safety considerations.

Grade 12 Essential Mathematics – Half Course V

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Statistics		General Learning Outcome: Develop statistical reasoning.
Specific Learning Outcomes It is expected that students will:		Achievement Indicators The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.
12E5.S.1.	Solve problems that involve measures of central tendency, including ■ mean ■ median ■ mode ■ weighted mean ■ trimmed mean [C, CN, PS, R]	■ Explain, using examples, the advantages and disadvantages of each measure of central tendency. ■ Determine the mean, median, and mode for a set of data. ■ Identify and correct errors in a calculation of a measure of central tendency. ■ Identify the outlier(s) in a set of data. ■ Explain the effect of outliers on mean, median, and mode. ■ Determine the trimmed mean for a set of data, and justify the removal of the outliers. ■ Explain, using examples such as course marks, why some data in a set would be given a greater weighting in determining the mean. ■ Determine the weighted mean of a set of data, and justify the different weightings. ■ Explain, using examples from print or other media, how measures of central tendency and outliers are used to provide different interpretations of data. ■ Solve a contextual problem that involves measures of central tendency.
12E5.S.2.	Analyze and describe percentiles. [C, CN, PS, R]	■ Explain, using examples, percentile ranks in a context. ■ Explain decisions based on a given percentile rank. ■ Explain, using examples, the difference between percent and percentile rank. ■ Explain the relationship between median and percentile. ■ Solve a contextual problem that involves percentiles. ■ Determine the percentile rank of a given score using a formula.

Grade 12 Essential Mathematics – Half Course V

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Precision Measurement

General Learning Outcome:
Develop spatial sense relating to the limitations of measuring instruments.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 12E5.P.1. Demonstrate an understanding of the limitations of measuring instruments, including
- precision
 - accuracy
 - uncertainty
 - tolerance
- [C, PS, R, T, V]

- Explain why, in a given context, a certain degree of precision is required.
- Explain why, in a given context, a certain degree of accuracy is required.
- Explain, using examples, the difference between precision and accuracy.
- Compare the degree of accuracy of two instruments used to measure the same attribute.
- Relate the degree of accuracy to the uncertainty of a given measure.
- Analyze precision and accuracy in a contextual problem.
- Determine maximum and minimum values, using a degree of tolerance in context.
- Describe, using examples, the limitations of measuring instruments such as tape measure, micrometer, or Vernier caliper, used in a specific trade or industry.
- Solve a problem that involves precision, accuracy, or tolerance.

Grade 12 Essential Mathematics – Half Course V

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Career Life

General Learning Outcome:
Develop a plan for the future.

Specific Learning Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12E5.C.1 Create a plan for the future, including possible career choices and their requirements.
[C, CN, PS, R]

- Describe factors important in selecting careers, including job description, educational requirements and costs, aptitude, values, salary/wage levels, employment opportunities, opportunities for advancement, and work/life balance.
- Prepare a monthly budget for the preparatory period for a chosen career.
- Prepare a monthly budget for the lifestyle achievable for a chosen career.
- Analyze the lifestyle one specific career could entail in terms of monthly budget, lifestyle appropriate to the career, and any negative factors of the job.
- Prepare a resumé for your career choice.

Grade 12 Essential Mathematics – Half Course VI

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Analysis of Games and Numbers

General Learning Outcome:
Develop critical-thinking skills.

Specific Learning Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 12E6.A.1. Analyze puzzles and games that involve logical reasoning, using problem-solving strategies.
[C, CN, PS, R]

It is intended that this learning outcome be integrated throughout the course by using puzzles and games such as Sudoku, Mastermind, Nim, and logic puzzles.

- Determine, explain, and verify a strategy to solve a puzzle or to win a game such as
 - guess and check
 - look for a pattern
 - make a systematic list
 - draw or model
 - eliminate possibilities
 - simplify the original problem
 - work backward
 - develop alternative approaches
- Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.
- Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Grade 12 Essential Mathematics – Half Course VI

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Home Finance

General Learning Outcome:
Develop an understanding of housing costs.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12E6.H.1. Solve problems involving the purchase and maintenance of a house.
[C, CN, ME, R, T]

- Solve a problem involving mortgages.
- Describe the costs involved in purchasing a home, such as closing costs, land transfer tax, lawyer's fees, house insurance, and moving expenses.
- Solve a problem involving home insurance options.
- Discuss the difference between preventative maintenance and emergency repair costs.
- Compare the benefits of owning and renting a house.
- Discuss energy efficiency options and the immediate and long-term impact on your housing costs.
- Discuss the daily costs involved with home maintenance.
- Determine the property tax for a house.
- Determine the economic feasibility of a home purchase using the Gross Debt Service Ratio.

Grade 12 Essential Mathematics – Half Course VI

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Geometry and Trigonometry		General Learning Outcome: Develop spatial sense involving polygons.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
12E6.G.1.	Solve problems by using the sine law and cosine law, excluding the ambiguous case. [CN, PS, V]	- Identify and describe the use of the sine law or cosine law in construction, industrial, commercial, or artistic applications. - Solve a problem, using the sine law or cosine law, when a diagram is given.
12E6.G.2.	Solve problems that involve - triangles - quadrilaterals - regular polygons [C, CN, PS, V]	- Describe and illustrate properties of triangles, including isosceles or equilateral. - Describe and illustrate properties of quadrilaterals in terms of angle measures, side lengths, diagonals, or angles of intersection. - Describe and illustrate properties of regular polygons. - Explain, using examples, why a given property does or does not apply to certain polygons. - Identify and explain an application of the properties of polygons in construction, industrial, commercial, domestic, or artistic contexts. - Solve a contextual problem that involves the application of the properties of polygons.

Grade 12 Essential Mathematics – Half Course VI

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Business Finance		General Learning Outcome: Develop an understanding of business finance.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
12E6.B.1.	Critique the viability of small business options by considering ■ expenses ■ sales ■ profit or loss [C, CN, R]	■ Identify expenses in operating a small business, such as a hot-dog stand or a lawn maintenance company. ■ Identify feasible small business options for a given community. ■ Generate options that might improve the profitability of a small business. ■ Determine the break-even point for a small business. ■ Explain factors, such as seasonal variations and hours of operation, that might affect the profitability of a small business.
12E6.B.2.	Demonstrate an awareness of the government taxation forms and procedures involved in owning a business. [C, CN]	■ Identify receipts needed to be retained for income tax forms. ■ Identify appropriate deductions that can be claimed under business expenses but not under personal expenses. ■ Identify the major sections of the T-1 General Income Tax Form and related schedules.

Grade 12 Essential Mathematics – Half Course VI

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Probability

General Learning Outcome:
Develop critical thinking skills related to uncertainty.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12E6.P.1. Analyze and interpret problems that involve probability.
[C, CN, PS, R]

- Describe and explain the applications of probability, such as medication, warranties, insurance, lotteries, weather prediction, 100-year flood, failure of a design, failure of a product, vehicle recalls, or approximation of area.
- Determine the probability of an event based on a data set.
- Express a given probability as a fraction, decimal, percent, or in a statement.
- Determine the expected gain or loss of a situation.
- Explain the difference between odds and probability.
- Determine the probability of an event, given the odds for or against.
- Explain, using examples, how decisions may be based on a combination of theoretical probability calculations, experimental results, and subjective judgments.
- Solve a contextual problem that involves a probability.

GRADE 12 PRE-CALCULUS MATHEMATICS (40S)

Grade 12 Pre-Calculus Mathematics (40S) is designed for students who intend to study calculus and related mathematics as part of post-secondary education. It builds on the topics studied in Grade 11 Pre-Calculus Mathematics and provides background knowledge and skills for the study of calculus in post-secondary institutions.

The course comprises a high-level study of theoretical mathematics with an emphasis on problem solving and mental mathematics. The topics include study of transformations of functions, trigonometric functions, exponential functions, logarithmic functions, polynomial functions, radical functions, rational functions, and the binomial theorem.

Assessment of Grade 12 Pre-Calculus Mathematics should be a balance of assessment *for* learning, assessment *as* learning, and assessment *of* learning. Assessment tools used in Grade 12 Pre-Calculus Mathematics should be varied and may include observation, homework, learning conversations or interviews, summative unit essays, demonstrations, presentations, performance tasks, learning logs, projects, investigations, reflective

journals, portfolios, quizzes, tests, and examinations. An appropriately prepared portfolio requires a consistent effort throughout the school term and a commitment to completing quality work on a daily basis.

The learning outcomes are divided into three topics: Trigonometry; Relations and Functions; and Permutations, Combinations, and Binomial Theorem. For instructional purposes, the learning outcomes could be arranged into units. Learning outcomes from different topics could be taught in the same unit. Some learning outcomes may fit into multiple units and parts of the learning outcome could be taught in one unit while the remaining parts can be taught later. Two possible sequences of the learning outcomes into units with suggested time allotments follow. The suggested times include time for instruction and assessment. These are not the only possibilities but will provide some direction for teachers for their first time through the course.

Regardless of the organization of the learning outcomes into units, students should constantly be looking for and be given opportunities to see connections between the various outcomes in Grade 12 Pre-Calculus Mathematics.

Possibility 1			Possibility 2		
Unit	Learning Outcomes	Suggested Hours	Unit	Learning Outcomes	Suggested Hours
Transformations of Functions	R1, R2, R3, R4, R5, R6	25	Functions	R11, R12, R13, R14	25
Trigonometric Functions	T1, T2, T3, T4	25	Transformations of Functions	R1, R2, R3, R4, R5, R6	25
Binomial Theorem	P1, P2, P3, P4	10	Trigonometric Functions	T1, T2, T3, T4, T5	28
Polynomial Functions	R11, R12	12	Permutations and Combinations	P1, P2, P3	5
Trigonometric Equations and Identities	T5, T6	15	Exponents and Logarithms	R7, R8, R9, R10	10
Exponents and Logarithms	R7, R8, R9, R10	10	Trigonometric Identities	T6	12
Radicals and Rationals	R13, R14	13	Binomial Theorem	P4	5
Total		110	Total		110

General and Specific Learning Outcomes with Achievement Indicators by Course

Grade 12 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Trigonometry		General Learning Outcome: Develop trigonometric reasoning.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
12P.T.1.	Demonstrate an understanding of angles in standard position, expressed in degrees and radians. [C, CN, ME, R, V]	■ Sketch, in standard position, an angle (positive or negative) when the measure is given in degrees. ■ Describe the relationship among different systems of angle measurement, with emphasis on radians and degrees. ■ Sketch, in standard position, an angle with a measure of 1 radian. ■ Sketch, in standard position, an angle with a measure expressed in the form $k\pi$ radians, where $k \in \mathbb{Q}$. ■ Express the measure of an angle in radians (exact value or decimal approximation), given its measure in degrees. ■ Express the measure of an angle in degrees, given its measure in radians (exact value or decimal approximation). ■ Determine the measures, in degrees or radians, of all angles in a given domain that are coterminal with an angle in standard position. ■ Determine the general form of the measures, in degrees or radians, of all angles that are coterminal with an angle in standard position. ■ Explain the relationship between the radian measure of an angle in standard position and the length of the arc cut on a circle of radius r, and solve a problem based upon that relationship.

Grade 12 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Trigonometry (<i>continued</i>)		General Learning Outcome: Develop trigonometric reasoning.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
12PT.2.	Develop and apply the equation of the unit circle. [CN, R, V]	- Derive the equation of the unit circle from the Pythagorean theorem. - Describe the six trigonometric ratios, using a point P (x, y) that is the intersection of the terminal arm of an angle and the unit circle. - Generalize the equation of a circle with centre (0, 0) and radius r.
12PT.3.	Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees. [C, ME, PS, R, T, V]	- Determine, with technology, the approximate value of a trigonometric ratio for any angle with a measure expressed in either degrees or radians. - Determine, using the unit circle or reference triangle, the exact value of a trigonometric ratio for angles expressed in degrees that are multiples of 0°, 30°, 45°, 60°, or 90°, or for angles expressed in radians that are multiples of 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, or $\frac{\pi}{2}$, and explain the strategy. - Determine, with or without technology, the measures, in degrees or radians, of the angles in a specified domain, given the value of a trigonometric ratio. - Explain how to determine the exact values of the six trigonometric ratios, given the coordinates of a point on the terminal arm of an angle in standard position. - Determine the measures of the angles in a specified domain in degrees or radians, given a point on the terminal arm of an angle in standard position. - Determine the exact values of the other trigonometric ratios, given the value of one trigonometric ratio in a specified domain. - Sketch a diagram to represent a problem that involves trigonometric ratios. - Solve a problem, using trigonometric ratios.

Grade 12 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Trigonometry (*continued*)

General Learning Outcome:
Develop trigonometric reasoning.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12P.T.4. Graph and analyze the trigonometric functions sine, cosine, and tangent to solve problems.
[C, CN, PS, T, V]

- Sketch, with or without technology, the graph of $y = \sin x$, $y = \cos x$, or $y = \tan x$.
- Determine the characteristics (amplitude, asymptotes, domain, period, range, and zeros) of the graph of $y = \sin x$, $y = \cos x$, or $y = \tan x$.
- Determine how varying the value of a affects the graphs of $y = a \sin x$ or $y = a \cos x$.
- Determine how varying the value of d affects the graphs of $y = \sin x + d$ or $y = \cos x + d$.
- Determine how varying the value of c affects the graphs of $y = \sin(x - c)$ or $y = \cos(x - c)$.
- Determine how varying the value of b affects the graphs of $y = \sin bx$ or $y = \cos bx$.
- Sketch, without technology, graphs of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$, using transformations, and explain the strategies.
- Determine the characteristics (amplitude, asymptotes, domain, period, phase shift, range, and zeros) of the graph of a trigonometric function of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$.
- Determine the values of a , b , c , and d for functions of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$ that correspond to a graph, and write the equation of the function.
- Determine a trigonometric function that models a context to solve a problem.
- Explain how the characteristics of the graph of a trigonometric function relate to the conditions in a problem context.
- Solve a problem by analyzing the graph of a trigonometric function.

Grade 12 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand: Trigonometry (<i>continued</i>)		General Learning Outcome: Develop trigonometric reasoning.
Specific Learning Outcomes <i>It is expected that students will:</i>		Achievement Indicators <i>The following set of indicators may be used to determine whether students have met the corresponding specific learning outcome.</i>
12P.T.5.	Solve, algebraically and graphically, first- and second-degree trigonometric equations with the domain expressed in degrees and radians. [C, CN, PS, R, T, V] Students should be able to solve first-degree sine, cosine, and tangent double-angle trigonometric equations.	■ Verify, with or without technology, that a value is a solution to a trigonometric equation. ■ Determine, algebraically, the solution of a trigonometric equation, stating the solution in exact form when possible. ■ Determine, using technology, the approximate solution of a trigonometric equation in a restricted domain. ■ Relate the general solution of a trigonometric equation to the zeros of the corresponding trigonometric function. ■ Determine, using technology, the general solution of a trigonometric equation. ■ Identify and correct errors in a solution for a trigonometric equation. ■ Use identities to simplify and solve a trigonometric equation.
12P.T.6.	Prove trigonometric identities, using ■ reciprocal identities ■ quotient identities ■ Pythagorean identities ■ sum or difference identities (restricted to sine, cosine, and tangent) ■ double-angle identities (restricted to sine, cosine, and tangent) [C, R, T, V]	■ Explain the difference between a trigonometric identity and a trigonometric equation. ■ Verify a trigonometric identity numerically for a given value in either degrees or radians. ■ Explain why verifying that the two sides of a trigonometric identity are equal for given values is insufficient to conclude that the identity is valid. ■ Determine, graphically, the potential validity of a trigonometric identity, using technology. ■ Determine the non-permissible values of a trigonometric identity. ■ Prove a trigonometric identity algebraically. ■ Determine, using the sum, difference, or double-angle identities, the exact value of a trigonometric ratio.

Grade 12 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions

General Learning Outcome:
Develop algebraic and graphical reasoning
through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12P.R.1. Demonstrate an understanding of operations on, and compositions of, functions.
[CN, R, T, V]

- Sketch the graph of a function that is the sum, difference, product, or quotient of two functions, given their graphs.
- Write the equation of a function that is the sum, difference, product, or quotient of two or more functions, given their equations.
- Determine the domain and range of a function that is the sum, difference, product, or quotient of two functions.
- Write a function $f(x)$ as the sum, difference, product, or quotient of two or more functions.
- Determine the value of the composition of functions when evaluated at a point using the forms $f(f(a))$, $f(g(a))$, or $g(f(a))$.
- Determine, given the equations of two functions $f(x)$ and $g(x)$, the equation of the composite function of the forms $f(f(x))$, $f(g(x))$, or $g(f(x))$, and explain any restrictions.
- Sketch, given the equations of two functions $f(x)$ and $g(x)$, the graph of the composite function in the forms $f(f(x))$, $f(g(x))$, or $g(f(x))$.
- Sketch the graph of the function $y = |f(x)|$ or $y = \frac{1}{f(x)}$ given the graph of $y = f(x)$, and explain the strategies used.
- Write a function $f(x)$ as the composition of two or more functions.
- Write a function $f(x)$ by combining two or more functions through operations on, or compositions of, functions.

Grade 12 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning
through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12P.R.2.	Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations. [C, CN, R, V]	■ Compare the graphs of a set of functions of the form $y - k = f(x)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of k. ■ Compare the graphs of a set of functions of the form $y = f(x - h)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of h. ■ Compare the graphs of a set of functions of the form $y - k = f(x - h)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effects of h and k. ■ Sketch the graph of $y - k = f(x)$, $y = f(x - h)$ or $y - k = f(x - h)$ for values of h and k, given a sketch of the function $y = f(x)$, where the equation of $y = f(x)$ is not given. ■ Write the equation of a function whose graph is a vertical or horizontal translation of the graph of the function.
----------	--	---

Grade 12 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions *(continued)*

General Learning Outcome:
Develop algebraic and graphical reasoning
through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

- 12P.R.3. Demonstrate an understanding of the effects of horizontal and vertical compressions and stretches on the graphs of functions and their related equations.
[C, CN, R, V]

A compression by a factor of a is the same as a stretch by a factor of $\frac{1}{a}$.

- Compare the graphs of a set of functions of the form $y = af(x)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of a .
- Compare the graphs of a set of functions of the form $y = f(bx)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effect of b .
- Compare the graphs of a set of functions of the form $y = af(bx)$ to the graph of $y = f(x)$, and generalize, using inductive reasoning, a rule about the effects of a and b .
- Sketch the graph of $y = af(x)$, $y = f(bx)$, or $y = af(bx)$ for values of a and b , given a sketch of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.
- Write the equation of a function, given its graph, which is a vertical or horizontal compression or stretch of the graph of the function $y = f(x)$.

- 12P.R.4. Apply translations, compressions, and stretches to the graphs and equations of functions.
[C, CN, R, V]

- Sketch the graph of the function $y - k = af(b(x - h))$ for values of a , b , h , and k , given the graph of the function $y = f(x)$, where the equation of $y = f(x)$ is not given.
- Write the equation of a function, given its graph which is a translation, compression, or stretch of the graph of the function $y = f(x)$.

Grade 12 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12P.R.5.	Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections through the ■ x-axis ■ y-axis ■ line $y = x$ [C, CN, R, V]	■ Generalize the relationship between the coordinates of an ordered pair and the coordinates of the corresponding ordered pair that results from a reflection through the x-axis, the y-axis, or the line $y = x$. ■ Sketch the reflection of the graph of a function $y = f(x)$ through the x-axis, the y-axis, or the line $y = x$, given the graph of the function $y = f(x)$, where the equation of $y = f(x)$ is not given. ■ Generalize, using inductive reasoning, and explain rules for the reflection of the graph of the function $y = f(x)$ through the x-axis, the y-axis, or the line $y = x$. ■ Sketch the graphs of the functions $y = -f(x)$, $y = f(-x)$, and $y = f^{-1}(x)$ given the graph of the function $y = f(x)$, where the equation of $y = f(x)$ is not given. ■ Write the equation of a function, given its graph, which is a reflection of the graph of the function through the x-axis, the y-axis, or the line $y = x$.
12P.R.6.	Demonstrate an understanding of inverses of relations. [C, CN, R, V]	- n Explain how the graph of the line $y = x$ can be used to sketch the inverse of a relation. ■ Explain how the transformation $(x, y) \rightarrow (y, x)$ can be used to sketch the inverse of a relation. ■ Sketch the graph of the inverse relation, given the graph of a relation. ■ Determine whether a relation and its inverse are functions. ■ Determine restrictions on the domain of a function in order for its inverse to be a function. ■ Determine the equation and sketch the graph of the inverse relation, given the equation of a linear or quadratic relation. ■ Explain the relationship between the domains and ranges of a relation and its inverse. ■ Determine, algebraically or graphically, whether two functions are inverses of each other.

Grade 12 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning
through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators
*The following set of indicators **may** be used to determine whether
students have met the corresponding specific learning outcome.*

12P.R.7. Demonstrate an understanding of logarithms.
[C, CN, ME, R]

- Explain the relationship between logarithms and exponents.
- Express a logarithmic expression as an exponential expression, and vice versa.
- Determine, without technology, the exact value of a logarithm.
- Estimate the value of a logarithm, using benchmarks, and explain the reasoning.

12P.R.8. Demonstrate an understanding of the product, quotient, and
power laws of logarithms.
[C, CN, R, T]

- Develop and generalize the laws for logarithms, using numeric examples and exponent laws.
- Prove each law of logarithms.
- Determine, using the laws of logarithms, an equivalent expression for a logarithmic expression.
- Determine, with technology, the approximate value of a logarithmic expression.

Grade 12 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12P.R.9. Graph and analyze exponential and logarithmic functions.
[C, CN, T, V]

It is intended that students will be able to work with logarithms of any base, $b > 1$, including base e .

- Sketch, with or without technology, a graph of an exponential function of the form $y = a^x$, $a > 0$.
- Identify the characteristics of the graph of an exponential function of the form $y = a^x$, $a > 0$, including the domain, range, horizontal asymptote, and intercepts, and explain the significance of the horizontal asymptote.
- Sketch the graph of an exponential function by applying a set of transformations to the graph of $y = a^x$, $a > 0$, and state the characteristics of the graph.
- Sketch, with or without technology, the graph of a logarithmic function of the form $y = \log_b x$, $b > 1$.
- Identify the characteristics of the graph of a logarithmic function of the form $y = \log_b x$, $b > 1$, including the domain, range, vertical asymptote, and intercepts, and explain the significance of the vertical asymptote.
- Sketch the graph of a logarithmic function by applying a set of transformations to the graph of $y = \log_b x$, $b > 1$, and state the characteristics of the graph.
- Demonstrate, graphically, that a logarithmic function and an exponential function with the same base are inverses of each other.

12P.R.10. Solve problems that involve exponential and logarithmic equations.
[C, CN, PS, R]

- Determine the solution of an exponential equation in which the bases are powers of one another.
- Determine the solution of an exponential equation in which the bases are not powers of one another, using a variety of strategies.
- Determine the solution of a logarithmic equation, and verify the solution.
- Explain why a value obtained in solving a logarithmic equation may be extraneous.
- Solve a problem that involves exponential growth or decay.
- Solve a problem that involves the application of exponential equations to loans, mortgages, or investments.
- Solve a problem that involves logarithmic scales, such as the Richter scale or the pH scale.
- Solve a problem by modelling a situation with an exponential or a logarithmic equation.

Grade 12 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning
through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12P.R.11. Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree ≤ 5 with integral coefficients).
[C, CN, ME]

- Explain how long division of a polynomial expression by a binomial expression of the form $x - a$, $a \in I$ is related to synthetic division.
- Divide a polynomial expression by a binomial expression of the form $x - a$, $a \in I$, using long division or synthetic division.
- Explain the relationship between the linear factors of a polynomial expression and the zeros of the corresponding polynomial function.
- Explain the relationship between the remainder when a polynomial expression is divided by $x - a$, $a \in I$ and the value of the polynomial expression at $x = a$ (remainder theorem).
- Explain and apply the factor theorem to express a polynomial expression as a product of factors.

12P.R.12. Graph and analyze polynomial functions (limited to polynomial functions of degree ≤ 5).
[C, CN, PS, T, V]

- Identify the polynomial functions in a set of functions, and explain the reasoning.
- Explain the role of the constant term and leading coefficient in the equation of a polynomial function with respect to the graph of the function.
- Generalize rules for graphing polynomial functions of odd or even degree.
- Explain the relationship between the zeros of a polynomial function, the roots of the corresponding polynomial equation, and the x -intercepts of the graph of the polynomial function.
- Explain how the multiplicity of a zero of a polynomial function affects the graph.
- Sketch, with or without technology, the graph of a polynomial function.
- Solve a problem by modelling a context with a polynomial function and analyzing the graph of the function.

Grade 12 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Relations and Functions (*continued*)

General Learning Outcome:
Develop algebraic and graphical reasoning through the study of relations.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12P.R.13. Graph and analyze radical functions (limited to functions involving one radical).
[C, CN, R, T, V]

- Sketch the graph of the function $y = \sqrt{x}$, using a table of values, and state the domain and range.
- Sketch the graph of the function $y - k = a\sqrt{b(x - h)}$ by applying transformations to the graph of the function $y = \sqrt{x}$, and state the domain and range.
- Sketch the graph of the function $y = \sqrt{f(x)}$, given the graph of the function $y = f(x)$, and explain the strategies used.
- Compare the domain and range of the function $y = \sqrt{f(x)}$, to the domain and range of the function $y = f(x)$, and explain why their domains and ranges may differ.
- Describe the relationship between the roots of a radical equation and the x-intercepts of the graph of the corresponding radical function.
- Determine, graphically, an approximate solution of a radical equation.

12P.R.14. Graph and analyze rational functions (limited to numerators and denominators that are monomials, binomials, or trinomials).
[C, CN, R, T, V]

- Graph, with or without technology, a rational function.
- Analyze the graphs of a set of rational functions to identify common characteristics.
- Explain the behaviour of the graph of a rational function for values of the variable near a non-permissible value.
- n Determine if the graph of a rational function will have an asymptote or a point of discontinuity (hole) for a non-permissible value.
- Match a set of rational functions to their graphs, and explain the reasoning.
- Describe the relationship between the roots of a rational equation and the x-intercepts of the graph of the corresponding rational function.
- Determine, graphically, an approximate solution of a rational equation.

Grade 12 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Permutations, Combinations and Binomial Theorem

General Learning Outcome:
Develop algebraic and numeric reasoning that involves combinatorics.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12P.P.1	Apply the fundamental counting principle to solve problems. [C, CN, PS, R, V]	- Count the total number of items in the sample space, using graphic organizers such as lists and tree diagrams. - Explain, using examples, why the total number of items is found by multiplying rather than adding the number of ways the individual choices can be made. - Solve a simple counting problem by applying the fundamental counting principle.
12P.P.2.	Determine the number of permutations of n elements taken r at a time to solve problems. [C, PS, R, V]	- Count, using graphic organizers such as lists and tree diagrams, the number of ways of arranging the elements of a set in a row. - Determine, in factorial notation, the number of permutations of n different elements taken n at a time to solve a problem. - Determine, using a variety of strategies, the number of permutations of n different elements taken r at a time to solve a problem. - Explain why n must be greater than or equal to r in the notation ${}_nP_r$. - Solve an equation that involves ${}_nP_r$ notation. - Explain, using examples, the effect on the total number of permutations when two or more elements are identical.

Students should be able to use strategies such as cases or grouping objects together to solve a contextual problem. Also, it is intended that circular permutations not be included.

Grade 12 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:

Permutations, Combinations and Binomial Theorem (*continued*)

General Learning Outcome:

Develop algebraic and numeric reasoning that involves combinatorics.

Specific Learning Outcomes

It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12P.P.3.	Determine the number of combinations of n different elements taken r at a time to solve problems. [C, PS, R, V]	■ Explain, using examples, the difference between a permutation and a combination. ■ Determine the number of combinations of n different elements taken r at a time to solve a problem. ■ Explain why n must be greater than or equal to r in the notation ${}_nC_r$ or $\binom{n}{r}$. ■ Explain, using examples, why ${}_nC_r = {}_nC_{n-r}$ or $\binom{n}{r} = \binom{n}{n-r}$. ■ Solve an equation that involves ${}_nC_r$ or $\binom{n}{r}$ notation.
----------	--	--

Grade 12 Pre-Calculus Mathematics

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[ME] Mental Mathematics and Estimation	[T] Technology
	[V] Visualization

Strand:
Permutations, Combinations and Binomial Theorem (*continued*)

General Learning Outcome:
Develop algebraic and numeric reasoning that involves combinatorics.

Specific Learning Outcomes
It is expected that students will:

Achievement Indicators

*The following set of indicators **may** be used to determine whether students have met the corresponding specific learning outcome.*

12P.P.4. Expand powers of a binomial in a variety of ways, including using the binomial theorem (restricted to exponents that are natural numbers).
[C, CN, R, V]

- Explain the patterns found in the expanded form of $(x + y)^n$, $n \leq 4$, by multiplying n factors of $(x + y)$.
- Explain how to determine the subsequent row in Pascal's triangle, given any row.
- Relate the coefficients of the terms in the expansion of $(x + y)^n$ to the $(n + 1)^{\text{st}}$ row in Pascal's triangle.
- Explain, using examples, how the coefficients of the terms in the expansion of $(x + y)^n$ are determined by combinations.
- Expand, using the binomial theorem $(x + y)^n$.
- Determine a specific term in the expansion of $(x + y)^n$.

REFERENCES

- Alberta Education. *The Alberta K–9 Mathematics: Program of Studies with Achievement Indicators*. Edmonton, AB: Alberta Education, 2007. Available online at <http://education.alberta.ca/media/645598/kto9math_ind.pdf>
- American Association for the Advancement of Science [AAAS-Benchmarks]. *Benchmark for Science Literacy*. New York, NY: Oxford University Press, 1993.
- Anderson, A.G. “Parents as Partners: Supporting Children’s Mathematics Learning Prior to School.” *Teaching Children Mathematics* 4.6 (February 1998): 331–337.
- Armstrong, Thomas. *Seven Kinds of Smart: Identifying and Developing Your Many Intelligences*. New York, NY: NAL-Dutton, 1993.
- Ashlock, R. “Diagnosing Error Patterns in Computation.” *Error Patterns in Computation*. Columbus, OH: Prentice Hall, 1998. 9–42.
- Banks, J.A., and C.A.M. Banks. *Multicultural Education: Issues and Perspectives*. Boston, MA: Allyn and Bacon, 1993.
- Battista, Michael T. *Cognition-Based Assessment and Teaching of Addition and Subtraction: Building on Students’ Reasoning*. Portsmouth, NH: Heinemann, 2012.
- Becker, J.P., and S. Shimada. *The Open-Ended Approach: A New Proposal for Teaching Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 1997.
- Ben-Chaim, David, Glenda Lappan, and Richard T. Houang. “Adolescents Ability to Communicate Spatial Information: Analyzing and Effecting Students’ Performance.” *Educational Studies Mathematics* 20.2 (May 1989): 121–146.
- Borasi, R. *Learning Mathematics through Inquiry*. Portsmouth, NH: Heinmann, 1992.
- . *Reconceiving Mathematics Instruction: A Focus on Errors*. Norwood, NJ: Ablex, 1996.
- Bright, George W., Wallece Brewer, Kay McClain, and Edward S. Mooney. *Navigating through Data Analysis in Grades 6–8*. Reston, VA: National Council of Teachers of Mathematics, 2003.
- British Columbia Ministry of Education. *The Primary Program: A Framework for Teaching*, Victoria, BC: British Columbia Ministry of Education, 2000.
- Burke, M.J., and F.R. Curcio. *Learning Mathematics for a New Century* (2000 yearbook). Reston, VA: National Council of Teachers of Mathematics, 2000.

Burke, M., D. Erickson, J. Lott, and M. Obert. *Navigating through Algebra in Grades 9–12*. Reston, VA: National Council of Teachers of Mathematics, 2001.

Burns, Marilyn. *About Teaching Mathematics: A K-8 Resource*. 2nd ed. Sausalito, CA: Math Solutions Publications, 2000.

Buschman, Larry. "Using Student Interviews to Guide Classroom Instruction: An Action Research Project." *Teaching Children Mathematics* (December 2001): 222–227.

Caine, Renate Numella, and Geoffrey Caine. *Making Connections: Teaching and the Human Brain*. Menlo Park, CA: Addison-Wesley Publishing Company, 1991.

Chambers, D.L., ed. *Putting Research into Practice in the Elementary Grades*. Reston, VA: National Council of Teachers of Mathematics, 2002.

Chapin, Suzanne, Alice Koziol, Jennifer MacPherson, and Carol Rezba. *Navigating through Data Analysis and Probability in Grades 3–5*. Reston VA: National Council of Teachers of Mathematics, 2003.

Charles, Randall, and Joanne Lobato. *Future Basics: Developing Numerical Power, a Monograph of the National Council of Supervisors of Mathematics*. Golden, CO: National Council of Supervisors of Mathematics, 1998.

Clements, D.H. "Geometric and Spatial Thinking in Young Children." *Mathematics in the Early Years*. Ed. J. Copley. Reston, VA: National Council of Teachers of Mathematics, 1999. 66–79.

———. "Subitizing: What is it? Why teach it?" *Teaching Children Mathematics* (March 1999): 400–405.

Colan, L., and J. Pegis. *Elementary Mathematics in Canada: Research Summary and Classroom Implications*. Toronto, ON: Pearson Education Canada, 2003.

Confrey, J. "A Review of the Research on Student Conceptions in Mathematics, Science and Programming." *Review of Research in Education* 16. Ed. C. Cadzen. Washington, DC: American Educational Research Association, 1990. 3–56.

Cuevas, G., and K. Yeatt. *Navigating through Algebra in Grades 3–5*. Reston VA: National Council of Teachers of Mathematics, 2001.

Dacey, Linda, Mary Cavanagh, Carol R. Findell, Carole E. Greenes, Linda Jensen Sheffield, and Marion Small. *Navigating through Measurement in Prekindergarten – Grade 2*. Reston, VA: National Council of Teachers of Mathematics, 2003.

Davis, R.B., and C.M. Maher. "What Do We Do When We 'Do Mathematics'?" *Constructivist Views on the Teaching and Learning of Mathematics*. Reston, VA: The National Council of the Teachers of Mathematics, 1990. 195–210.

- Day, Roger, Paul Kelley, Libby Krussel, Johnny Wilott, and James Hirstein. *Navigating through Geometry in Grades 9–12*. Reston VA: National Council of Teachers of Mathematics, 2002.
- Egan, K. *The Educated Mind: How Cognitive Tools Shape our Understanding*. Chicago, IL: University of Chicago Press, 1997.
- Fennell, F., and T. E. Landis. "Number Sense and Operations Sense." *Windows of Opportunity: Mathematics for Students with Special Needs*. Eds. C. A. Thornton and N. S. Blay. Reston, VA: National Council of Teachers of Mathematics, 1994. 187–203.
- Findell, C., M. Small, M. Cavanagh, L. Dacey, C. Greenes, and L. Sheffield. *Navigating through Geometry in Prekindergarten – Grade 2*. Reston, VA: National Council of Teachers of Mathematics, 2001.
- Friel, S., S. Rachlin, and D. Doyle. *Navigating through Algebra in Grades 6–8*. Reston, VA: National Council of Teachers of Mathematics, 2001.
- Fuys, D., D. Geddes, and R. Tischler. *The van Hiele Model of Thinking in Geometry among Adolescents*. Reston, VA: National Council of Teachers of Mathematics, 1998.
- Gattegno, C. *The Common Sense of Teaching Mathematics*. New York, NY: Educational Solutions, 1974.
- Gavin, M., L. Belkin, A. Spinelli, and J. St. Marie. *Navigating through Geometry in Grades 3–5*. Reston, VA: National Council of Teachers of Mathematics, 2001.
- Gay, S., and M. Thomas. "Just Because They Got It Right, Does it Mean They Know It?" *Assessment in the Mathematics Classroom*. Ed. N.L. Webb. Reston, VA: National Council of Teachers of Mathematics, 1993. 130–134.
- Ginsburg, H.P., L.S. Lopez Choi, R. Netley, and C. Chao-Yuan. "Happy Birthday to You: Early Mathematical Thinking of Asian, South American, and U.S. Children." *Learning and Teaching Mathematics: An International Perspective*. Eds. T. Nunes and P. Bryant. Hove, East Sussex: Psychology Press, 1997. 163–207.
- Greenes, C.M., M. Cavanagh, L. Dacey, C. Findell, and M. Small. *Navigating through Algebra in Prekindergarten – Grade 2*. Reston, VA: National Council of Teachers of Mathematics, 2001.
- Greeno, J. "Number sense as a situated knowing in a conceptual domain." *Journal for Research in Mathematics Education* 22.3 (1991): 170–218.
- Griffin, S. "Teaching Number Sense." *ASCD Educational Leadership* (February 2004): 39–42.
- Haylock, Derek, and Anne Cockburn. *Understanding Mathematics in the Lower Primary Years*. Thousand Oaks, CA: SAGE Publications Inc., 2003.

- Heaton, R.M. *Teaching Mathematics to the New Standards: Relearning the Dance*. New York, NY: Teachers College Press, 2001.
- Hope, Jack A., Larry Leutzing, Barbara J. Reys, and Robert E. Reys. *Mental Math in the Primary Grades*. New York, NY: Dale Seymour Publications, 1988.
- Hopkins, Ros, ed. *Early Numeracy in the Classroom*. Melbourne, AU: State of Victoria, 2001.
- Howden, H. "Teaching Number Sense." *Arithmetic Teacher* 36.6 (1989): 6–11.
- Howe, R. "Knowing and Teaching Elementary Mathematics." *Journal of Research in Mathematics Education* 30.5 (1999): 556–558.
- Hunting, R.P. "Clinical Interview Methods in Mathematics Education Research and Practice." *Journal of Mathematical Behavior* 16.2 (1997): 145–165.
- Kamii, C. *Multidigit Division – Two Teachers Using Piaget's Theory*. Colchester, VT: Teachers College Press, 1990.
- Kamii, C., and A. Dominick. "To Teach or Not to Teach Algorithms." *Journal of Mathematical Behavior* 16.1 (1997): 51–61.
- Kelly, A.G. "Why Can't I See the Tree? A Study of Perspective." *Teaching Children Mathematics* 9.3 (October 2002): 158–161.
- Kersaint, G. "Raking Leaves – The Thinking of Students." *Mathematics Teaching in the Middle School* 9.3 (November 2002): 158–161.
- Kilpatrick, J., J. Swafford, and B. Findell, eds. *Adding it Up: Helping Children Learn Mathematics*. Washington, DC: National Academy Press, 2001.
- Kilpatrick, J., W.G. Martin, and D. Schifter, eds. *A Research Companion to Principles and Standards for School Mathematics*, Reston, VA: National Council of Teachers of Mathematics, 2003.
- King, J. *The Art of Mathematics*. New York, NY: Fawcett Columbine, 1992.
- Lakoff, G., and R.E. Nunez. *Where Mathematics Comes From – How the Embodied Mind Brings Mathematics into Being*. New York, NY: Basic Books, 2000.
- Lampert, M. *Teaching Problems and the Problems of Teaching*. New Haven, CT: Yale University Press, 2001.
- Ma, L. *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*. Mahwah, NJ: Lawrence Erlbaum, 1999.
- Manitoba Education, Citizenship and Youth. *Communicating Student Learning: Guidelines for Schools 2008*. Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2008.

- . *Kindergarten to Grade 8 Mathematics: Manitoba Curriculum Framework of Outcomes*. Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2008.
- . *Rethinking Classroom Assessment with Purpose in Mind: Assessment for Learning, Assessment as Learning, Assessment of Learning*. Winnipeg, MB: Manitoba Education, Citizenship and Youth, 2006.
- Mann, R. *Balancing Act: The Truth behind the Equals Sign. Teaching Children Mathematics* (September 2004): 65–69.
- Martine, S.L., and J. Bay-Williams. “Investigating Students’ Conceptual Understanding of Decimal Fractions.” *Mathematics Teaching in the Middle School* 8.5 (January 2003): 244–247.
- McAskill, Bruce, Glen Holmes, Leslee Francis-Pelton, and Wayne Watt. *WNCP Mathematics Research Project: Final Report*. Victoria, BC: Holdfast Consultants Inc., 2004.
- McAskill, B., G. Holmes, and L. Francis-Pelton. *Consultation Draft for the Common Curriculum Framework Kindergarten to Grade 9 Mathematics*. Victoria, BC: Holdfast Consultants Inc., 2005.
- Moran, G.J.W. *Identifying the van Hiele Levels of Geometry Thinking in Seventh-Grade Students through the Use of Journal Writing*. Diss. University of Massachusetts, Abstracts International 54.02 (1993): 464A.
- National Council of Teachers of Mathematics. *Computation, Calculators, and Common Sense*. Reston, VA: National Council of Teachers of Mathematics, May 2005.
- . *Problem Solving*. 2012. <www.nctm.org/standards/content.aspx?id=26860>. (26 Nov. 2012).
- . *The Teaching and Learning of Algorithms in School Mathematics*, 1998 Yearbook. Lorna J. Morrow (1998 Yearbook Ed.) and Margaret J. Kenney (General Yearbook Ed.). Reston, VA: National Council of Teachers of Mathematics, 1998.
- National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO). *Common Core Standards Initiative. Common Core State Standards for Mathematics*. United States: NGA Center and CCSSO, 2010. Available online at <www.corestandards.org/assets/CCSSI_Math%20Standards.pdf>.
- National Research Council. *Adding It Up: Helping Children Learn Mathematics*. Jeremy Kilpatrick, Jane Swafford, and Bradford Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral Sciences and Education. Washington, DC: National Academy Press, 2001.
- Nelson-Thomson. *Mathematics Education: A Summary of Research, Theories, and Practice*. Scarborough, ON: Nelson, 2002.

OECD (Organisation for Economic Co-operation and Development). *PISA 2012 Field Trial Problem Solving Framework*. September 30, 2012. <www.oecd.org/pisa/pisaproducts/46962005.pdf>.

Ontario. Ministry of Education. *The Ontario Curriculum Grades 1–8: Mathematics*. Ontario: Ministry of Education, 2005. Available online at <www.edu.gov.on.ca/eng/curriculum/elementary/math18curr.pdf>.

Pape, S.J., and M.A. Tchshanov. “The Role of Representation(s) in Developing Mathematical Understanding.” *Theory into Practice* 40.2 (Spring 2001): 118–127.

Paulos, J. *Innumeracy: Mathematical Illiteracy and its Consequences*. New York, NY: Vintage Books, 1998.

Peck, D., S. Jencks, and M. Connell. “Improving Instruction through Brief Interviews.” *Arithmetic Teacher*, 37.3 (1989): 15–17.

Pepper, K.L., and R.P. Hunting. “Preschoolers’ Counting and Sharing.” *Journal for Research in Mathematics Education* 28.2 (March 1998): 164–183.

Peressini D., and J. Bassett. “Mathematical Communication in Students’ Responses to a Performance-Assessment Task.” *Communication in Mathematics K–12 and Beyond*. Ed. P.C. Elliot. Reston, VA: National Council of Teachers of Mathematics, 1996. 146–158.

Perry, J.A., and S.L. Atkins. “It’s Not Just Notation: Valuing Children’s Representations.” *Teaching Children Mathematics*. 9.1 (September 2002): 196–201.

Prince Edward Island. Department of Education. *Mental Math: Fact Learning, Mental Computation, Estimation – Grade 3 Teacher’s Guide*. Charlottetown, PE: Prince Edward Island Department of Education, 2008. Available online at <www.gov.pe.ca/photos/original/eecd_gr3math.pdf>.

Pugalee, D., J. Frykholm, A. Johnson, M. Slovin, C. Malloy, and R. Preston. *Navigating Through Geometry in Grades 6–8*. Reston, VA: National Council of Teachers of Mathematics, 2002.

Québec. Ministère de l’Éducation. *Québec Education Program: Preschool Education, Elementary Education*. Quebec: Government of Quebec, 2001. Available online at <www.mels.gouv.qc.ca/DGFJ/dp/programme_de_formation/primaire/educprg2001h.htm>.

Rigby-Heinemann. *First Steps in Mathematics: Number*. Sydney, AU: Regby-Heinemann, 2004.

Robitaille, D., G. Orpwood, and A. Taylor. *The TIMSS-Canada Report, Vol. 2–G4*. Vancouver, BC: Dept. of CUST – UBC, 1997.

Romagnano, L. *Wrestling with Change – The Dilemmas of Teaching Mathematics*. Portsmouth, NH: Heinemann, 1994.

- Rubenstein, Rheta N. *Mental Mathematics beyond the Middle School: Why? What? How?* 94.6 (September 2001): 442.
- Sakshaug, L., M. Olson, and J. Olson. *How much film? Children are mathematical problem solvers*. Reston, VA: National Council of Teachers of Mathematics, 2002. 17–20.
- Sawyer, W.W. *Mathematician's Delight*. New York, NY: Penguin Books, 1943. Cited in Moran, G.J.W., 1993.
- Seymour, Dale. *Mental Math in the Primary Grades*. Palo Alto, CA: Dale Seymour Publications, 1998.
- Shaw, J.M., and M.F.P. Cliatt. "Developing Measurement Sense." *New Directions for Elementary School Mathematics*. Ed. P.R. Trafton. Reston, VA: National Council of Teachers of Mathematics, 1989. 149–155.
- Sheffield, Linda Jensen, Mary Cavanagh, Linda Darcey, Carol R. Findell, Carol Greenes, and Marion Small. *Navigating through Data Analysis and Probability in Prekindergarten – Grade 2*. Reston, VA: National Council of Teachers of Mathematics, 2002.
- Singapore. Ministry of Education. Curriculum Planning and Development Division. *Mathematics Syllabus: Primary*. Singapore: Ministry of Education, 2006. Available online at <www.moe.gov.sg/education/syllabuses/sciences/files/maths-primary-2007.pdf>.
- Small, M. *PRIME: Patterns and Algebra*. Toronto, ON: Nelson Publishing, 2005.
- . *PRIME: Number and Operations*. Toronto, ON: Nelson Publishing, 2005.
- Solomon, Pearl Gold. *The Math We Need to "Know" and "Do."* Thousand Oaks, CA: Sage Publications, 2001.
- Steen, L.A., ed. *On the Shoulders of Giants – New Approaches to Numeracy*. Washington, DC: National Research Council, 1990.
- Stiff, Lee. "Constructivist Mathematics and Unicorns (President's Message)." *NCTM News Bulletin* July/August 2001: 3.
- Swarthout, M. "Average Days of Spring – Problem Solvers." *Teaching Children Mathematics* 8.7 (March 2002): 404–406.
- Tang, E.P., and H.P. Ginsburg. "Young Children's Mathematical Reasoning – A Psychological View." *Developing Mathematical Reasoning in Grades K–12*. Eds. L. Stiff and F. Curcio. Reston, VA: National Council of Teachers of Mathematics, 1999. 45–61.
- Teppo, Anne R. *Reflecting on NCTM's Principles and Standards in Elementary and Middle School Mathematics*. Preston, VA: National Council of Teachers of Mathematics, 2002.
- Van de Walle, John A. *Elementary and Middle School Mathematics: Teaching Developmentally*. 5th ed. Boston, MA: Pearson Education, Inc., 2004.

- Van de Walle, John A., Sandra Folk, Karen S. Karp, and Jennifer M. Bay-Williams. *Elementary and Middle School Mathematics: Teaching Developmentally*. 3rd Can. ed. Toronto, ON: Pearson Canada Inc., 2011.
- Van den Heuvel-Panhuizen, M., and Gravemeijer. "Tests Aren't All Bad – An Attempt to Change the Face of Written Tests in Primary School Mathematics Instruction." *Realistic Mathematics Education in Primary School: On the Occasion of the Opening of the Freudenthal Institute*. Ed. L. Streefland. Utrecht, Netherlands: CD-B Press, 1991. 54–64.
- Van Hiele, P.M. *Structure and Insight: A Theory of Mathematics Education*. Orlando, FL: Academic Press, 1986.
- Vygotsky, L.S. *Thought and Language*. Cambridge, MA: MIT Press, 1986.
- . *Mind in Society: The Development of Higher Psychological Processes*. Cambridge, MA: Harvard University Press, 1978.
- Western and Northern Canadian Protocol. *The Common Curriculum Framework for Grades 10–12 Mathematics: Western and Northern Canadian Protocol*. Edmonton, AB: Alberta Education, 2008.
- . *The Common Curriculum Framework for K–9 Mathematics: Western and Northern Canadian Protocol*. Edmonton, AB: Alberta Education, 2006.
- . *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings*, 2006. Available online at <www.wncp.ca/english/subjectarea/mathematics/historicaldocuments.aspx>.
- Willoughby, Steven. *Mathematics Education for a Changing World*. Alexandria, VA: Association of Supervision and Curriculum Development, 1990.
- Wright, R.J. Martland, A.K. Stafford, and G. Stanger. *Teaching Number*, London, UK: Paul Chapman, 2002.



Printed in Canada
Imprimé au Canada